

Lesson 3. Sensitivity Analysis, MathProg and GUSEK, Classification of Optimization Models

1 From last time...

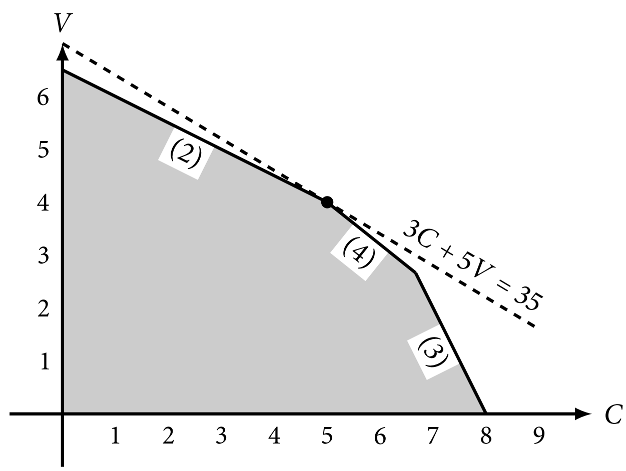
C = number of chocolate cakes to bake

V = number of vanilla cakes to bake

$$
\begin{aligned}
\text{maximize} \quad & 3C + 5V && (1)\\
\text{subject to} \quad & 20C + 40V \le 260 && (2)\\
& 4C + 2V \le 32 && (3)\\
& 4C + 5V \le 40 && (4)\\
& C \ge 0,\ V \ge 0 && (5)
\end{aligned}
$$

2 Sensitivity analysis

- For what profit margins on vanilla cakes will the current optimal solution remain optimal?



- Key observation:

[ ]

- Slope of (2) = [ ], slope of (4) = [ ]
- Let a be the new profit margin on vanilla cakes

  ⇒ objective function is [ ], slope of contour plots = [ ]

⇒ If [ ], then the current optimal solution remains optimal

3 Solving optimization models using a computer: MathProg and GUSEK

- To install and run GUSEK:
  - Download zip file from here:

    `http://sourceforge.net/projects/gusek/files/latest/download`
  - Unzip the file to any folder
  - Open the folder, double-click on `gusek.exe`
- Let's solve Farmer Jones's problem
  - Start a new file (if one isn't open already)
  - Type in the following MathProg code

```
# Define decision variables and variable bounds
var C >= 0;
var V >= 0;

# Objective function
maximize total_profit: 3*C + 5*V;

# General constraints
subject to baking: 20*C + 40*V <= 260;
subject to eggs: 4*C + 2*V <= 32;
subject to flour: 4*C + 5*V <= 40;

end;
```

  - Save as `farmerjones.mod` (`.mod` is the usual extension for MathProg code)
  - Make sure `Tools` → Generate Output File on Go is checked
  - Select Tools → Go
  - If all is well, a window on the left with the output file (`farmerjones.out`) will appear, and a log will appear on the right
- In the output file:
  - `STATUS` tells you if the model has an optimal solution, is unbounded, or is infeasible
  - `Objective` tells you the optimal value, if it exists
  - The table with `Column name` and `Activity` tells you the optimal solution
- MathProg notes:
  - Objective functions and constraints require **names**
    - Use something short and descriptive; no spaces allowed
  - End every statement with a semi-colon!
- Try solving Farmer Jones's problem with different profit margins for vanilla cakes. Does our sensitivity analysis match up with what you see?

4 Classification of optimization models

- Based on characteristics of
  - decision variables
  - constraints
  - objective function
- Decision variables can be continuous or integral
  - **Continuous**: can take on any value in a specified interval, e.g. $[0, +\infty)$
  - **Integral**: restricted to a specified interval of integers, e.g. $\{0, 1\}$
- Functions can be linear or nonlinear
  - A function $f(x_1, \dots, x_n)$ is **linear** if it is a constant-weighted sum of $x_1, \dots, x_n$; i.e.

    $$f(x_1, \dots, x_n) = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

    where $c_1, \dots, c_n$ are constants
  - Otherwise, a function is **nonlinear**
- Are these functions linear or nonlinear?
  - $f(x_1, x_2, x_3) = 9x_1 - 17x_3$
  - $f(x_1, x_2, x_3) = \dfrac{5}{x_1} + 3x_2 - 6x_3$
  - $f(x_1, x_2, x_3) = \dfrac{x_1 - x_2}{x_2 + x_3}$
  - $f(x_1, x_2, x_3) = x_1 x_2 + 3x_3$
- Constraints can be linear or nonlinear
  - A constraint can be written in the form

    $$g(x_1, \dots, x_n) \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b \qquad (*)$$

    where $g(x_1, \dots, x_n)$ is a function of decision variables $x_1, \dots, x_n$ and b is a specified constant
  - Constraint $(*)$ is **linear** if $g(x_1, \dots, x_n)$ is linear and **nonlinear** otherwise
- Strict inequalities (< or >) are not allowed in the optimization models we study

- An optimization model is a **linear program (LP)** if
  - the decision variables are continuous
  - the objective function is linear, and
  - the constraints are linear
- Are these optimization models linear programs?
  - $$\begin{aligned} \min \quad & 12z_1 + 4z_3 \\ \text{s.t.} \quad & z_1 z_2 z_3 = 1 \\ & z_1, z_2 \geq 0 \end{aligned}$$
  - $$\begin{aligned} \max \quad & 3z_1 + 14z_2 + 7z_3 \\ \text{s.t.} \quad & 10z_1 + 5z_2 \leq 25 - 18z_3 \\ & z_1 \geq 0, z_2 \geq 0, z_3 \geq 0 \end{aligned}$$
  - $$\begin{aligned} \min \quad & 3w_1 + 14w_2 - w_3 \\ \text{s.t.} \quad & 3w_1 + w_2 \leq 1 \\ & w_1 + 2w_2 + w_3 = 10 \\ & w_1 \geq 0, w_3 \geq 0 \\ & w_1 \text{ integer} \end{aligned}$$
  - Farmer Jones's model
- There are other types of optimization models: e.g. nonlinear programs, integer programs
- We may touch upon these later in the semester
- This semester, we will focus on linear programs