Lesson 5. Work Scheduling Models

Problem 2.3 from Rader A film packaging plant can manufacture four different thicknesses (1, 3, 5, and 0.5 mm) in any combination. Each thickness requires time on each of the three machines in minutes per square yard of film, as shown in the table below. Each machine is available 60 hours per week. The table also gives revenue and cost per square yard for each thickness. Variable labor costs are \$25 per hour for machines 1 and 2, and \$35 per hour for machine 3. Formulate and solve a profit-maximizing LP model for this problem, given the maximum demands for each thickness.

	Time (min)					
Thickness	1	2	3	Max Demand	Revenue	Cost
1 mm	5	8	9	400	110	30
3 mm	4	7	5	250	90	10
5 mm	4	5	4	200	60	10
0.5mm	6	10	6	450	100	20

Let x_1 = sq. yards of 1mm film to produce
x_2 = " " " 3mm " " "
x_3 = " " " 5mm " " "
x_4 = " " " 0.5mm " " "

y_1 = hours machine 1 used
y_2 = " " 2 "
y_3 = " " 3 "

max $110x_1 + 90x_2 + 60x_3 + 100x_4$ (revenue)
$-(30x_1 + 10x_2 + 10x_3 + 20x_4)$ (cost)
$-\frac{25}{60}(5x_1 + 4x_2 + 4x_3 + 6x_4)$
$-\frac{25}{60}(8x_1 + 7x_2 + 5x_3 + 10x_4)$
$-\frac{35}{60}(9x_1 + 5x_2 + 4x_3 + 6x_4)$
(variable labor cost)

can enter this directly w/o simplification in MathProg

s.t. $5x_1 + 4x_2 + 4x_3 + 6x_4 \leq 3600$ (machine 1)
$8x_1 + 7x_2 + 5x_3 + 10x_4 \leq 3600$ (machine 2)
$9x_1 + 5x_2 + 4x_3 + 6x_4 \leq 3600$ (machine 3)
$x_1 \leq 400$ (1mm film demand)
$x_2 \leq 250$ (3mm " ")
$x_3 \leq 200$ (5mm " ")
$x_4 \leq 450$ (0.5mm " ")
$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$

can enter these as constraints OR variable bounds:

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var x1 >= 0, <= 400
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max $110x_1 + 90x_2 + 60x_3 + 100x_4$
$-(30x_1 + 10x_2 + 10x_3 + 20x_4)$
$-\frac{25}{60}y_1 - \frac{25}{60}y_2 - \frac{35}{60}y_3$

s.t. $y_1 \leq 3600$, $y_2 \leq 3600$, $y_3 \leq 3600$
$x_1 \leq 400$, $x_2 \leq 250$, $x_3 \leq 200$,
$x_4 \leq 250$
$x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$

$$\boxed{\begin{aligned} y_1 &= 5x_1 + 4x_2 + 4x_3 + 6x_4 \\ y_2 &= 8x_1 + 7x_2 + 5x_3 + 10x_4 \\ y_3 &= 9x_1 + 5x_2 + 4x_3 + 6x_4 \end{aligned}}$$

x's and y's are related! Just because you define something doesn't make it so! Need to put the mathematical relationships in as constraints!

Problem. Postal employees in Simplexville work for 5 consecutive days, followed by 2 days off, repeated weekly. Below are the minimum number of employees needed for each day of the week:

Day	Employees needed
Monday (1)	7
Tuesday (2)	8
Wednesday (3)	7
Thursday (4)	6
Friday (5)	6
Saturday (6)	4
Sunday (7)	5

Write a linear program that determines the minimum total number of employees needed.

Hint. Define the following decision variables:

x_1 = number of employees who start work on Monday and work though Friday

x_2 = number of employees who start work on Tuesday and work though Saturday

$\vdots$

x_7 = number of employees who start work on Sunday and work through Thursday

Last time: If we define decision variables

y_1 = number of employees working on Monday

y_2 = " " " " " Tuesday

$\vdots$

y_7 = number of employees working on Sunday

Then writing linear constraints that ensure the right numbers of employees are working each day is straightforward:

$$y_1 \geq 7, \quad y_2 \geq 8, \quad y_3 \geq 7, \quad y_4 \geq 6, \quad y_5 \geq 6, \quad y_6 \geq 4, \quad y_7 \geq 5$$

BUT writing linear constraints that ensure employees work 5 consecutive days and 2 days off... ?