Lesson 12. Blending Models, Revisited

Problem 1. You are a portfolio manager in charge of a bank portfolio with at most \$10 million to invest. You want to maximize the earnings of your portfolio. There are 5 different securities available:

Bond name	Bond type	Quality Rating	Years to maturity	Yield at maturity
1	Municipal	2	9	4.3%
2	Agency	2	15	2.7
3	Gov't	1	4	2.5
4	Gov't	1	3	2.2
5	Municipal	5	2	4.5

The bank has some policies that limit how you can construct your portfolio:

1. The average quality of the portfolio cannot exceed 1.4 (lower quality rating = better)
2. The average years to maturity of the portfolio must be between 4 and 6 years
3. Bonds cannot be "shorted" (cannot buy negative amounts of bonds)

Describe the input parameters of this problem using sets and for statements.

Let B = set of bonds
t_i = type for bond i for $i \in B$
q_i = quality rating for bond i for $i \in B$
y_i = years to maturity for bond i for $i \in B$
p_i = yield at maturity for bond i for $i \in B$

C = amt. available to invest (in millions)
q^{max} = upper bound on avg. quality
y^{min} = lower bound on avg. years to maturity
y^{max} = upper bound on avg. years to maturity

Write a linear program for this problem using the symbolic input parameters you described above.

Decision variables: let x_i = amt invested in bond i (in millions) for $i \in B$

$$\max \sum_{i \in B} p_i x_i \quad \text{(total earnings)}$$

$$\text{s.t.} \quad \sum_{i \in B} x_i \le C \quad \text{(available funds)}$$

(✱)
$$\sum_{i \in B} q_i x_i \le q^{max} \sum_{i \in B} x_i \quad \text{(limits on avg. quality)}$$

$$y^{min} \sum_{i \in B} x_i \le \sum_{i \in B} y_i x_i \le y^{max} \sum_{i \in B} x_i \quad \text{(limits on avg. years to maturity)}$$

$$x_i \ge 0 \quad \text{for } i \in B \quad \text{(no shorting)}$$

$$\text{avg. quality} = \begin{pmatrix}\text{fraction in}\\ \text{bond 1}\end{pmatrix}\begin{pmatrix}\text{bond 1}\\ \text{quality}\end{pmatrix} + \begin{pmatrix}\text{fraction in}\\ \text{bond 2}\end{pmatrix}\begin{pmatrix}\text{bond 2}\\ \text{quality}\end{pmatrix} + \cdots$$

$$= \sum_{j \in B} \underbrace{\left(\frac{x_j}{\sum_{i \in B} x_i}\right)}_{\text{fraction in bond } j} q_j = \frac{\sum_{j \in B} q_j x_j}{\sum_{i \in B} x_i} = \underbrace{\frac{\sum_{i \in B} q_i x_i}{\sum_{i \in B} x_i}}_{\text{This needs to be } \le q^{max}}$$

Write a model and data file in MathProg for your linear program. Solve the linear program. What is the optimal value? What is the optimal solution?

optimal soln: $x_1 = 4,\ x_2 = 0,\ x_3 = 6,\ x_4 = 0,\ x_5 = 0$

optimal value: 0.322

Bonus. Can you make the linear program we wrote even more general? What sets and input parameters would you have to change, add, or delete?

For example, $P = \{\text{quality, years to maturity}\}$

Let P = set of properties

a_{ij} = property j for bond i

l_j = lower bound on average property j

u_j = upper bound on average property j

Replace ⊛ with: $l_j \sum_{i \in B} x_i \leq \sum_{i \in B} a_{ij} x_i \leq u_j \sum_{i \in B} x_i \quad \text{for } j \in P$