

Lesson 12. Blending Models, Revisited

Problem 1. You are a portfolio manager in charge of a bank portfolio with at most \$10 million to invest. You want to maximize the earnings of your portfolio. There are 5 different securities available:

Bond name	Bond type	Quality Rating	Years to maturity	Yield at maturity
1	Municipal	2	9	4.3%
2	Agency	2	15	2.7
3	Gov't	1	4	2.5
4	Gov't	1	3	2.2
5	Municipal	5	2	4.5

The bank has some policies that limit how you can construct your portfolio:

1. The average quality of the portfolio cannot exceed 1.4 (lower quality rating = better)
2. The average years to maturity of the portfolio must be between 4 and 6 years
3. Bonds cannot be “shorted” (cannot buy negative amounts of bonds)

Describe the input parameters of this problem using sets and for statements.

Let $B =$ set of bonds	$C =$ amt. available to invest (in millions)
$t_i =$ type for bond i for $i \in B$	$q^{\max} =$ upper bound on avg. quality
$q_i =$ quality rating for bond i for $i \in B$	$y^{\min} =$ lower bound on avg. years to maturity
$y_i =$ years to maturity for bond i for $i \in B$	$y^{\max} =$ upper bound on avg. years to maturity
$p_i =$ yield at maturity for bond i for $i \in B$	

Write a linear program for this problem using the symbolic input parameters you described above.

Decision variables: let $x_i =$ amt invested in bond i (in millions) for $i \in B$

max $\sum_{i \in B} p_i x_i$ (total earnings)

s.t. $\sum_{i \in B} x_i \leq C$ (available funds)

$\left\{ \begin{array}{l} \sum_{i \in B} q_i x_i \leq q^{\max} \sum_{i \in B} x_i \text{ (limits on avg. quality)} \\ y^{\min} \sum_{i \in B} x_i \leq \sum_{i \in B} y_i x_i \leq y^{\max} \sum_{i \in B} x_i \text{ (limits on avg. years to maturity)} \\ x_i \geq 0 \text{ for } i \in B \text{ (no shorting)} \end{array} \right.$

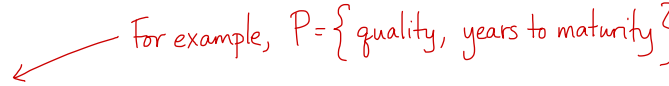
$\text{avg. quality} = \left(\frac{\text{fraction in bond 1}}{\text{bond 1}} \right) (\text{quality}) + \left(\frac{\text{fraction in bond 2}}{\text{bond 2}} \right) (\text{quality}) + \dots$
 $= \sum_{j \in B} \left(\frac{x_j}{\sum_{i \in B} x_i} \right) q_j = \frac{\sum_{j \in B} q_j x_j}{\sum_{i \in B} x_i} = \frac{\sum_{i \in B} q_i x_i}{\sum_{i \in B} x_i}$
 This needs to be $\leq q^{\max}$

Write a model and data file in MathProg for your linear program. Solve the linear program. What is the optimal value? What is the optimal solution?

optimal soln: $x_1=4, x_2=0, x_3=6, x_4=0, x_5=0$

optimal value: 0.322

Bonus. Can you make the linear program we wrote even more general? What sets and input parameters would you have to change, add, or delete?

Let $P =$ set of properties  For example, $P = \{ \text{quality, years to maturity} \}$
 $a_{ij} =$ property j for bond i
 $l_j =$ lower bound on average property j
 $u_j =$ upper bound on average property j

Replace \otimes with: $l_j \sum_{i \in B} x_i \leq \sum_{i \in B} a_{ij} x_i \leq u_j \sum_{i \in B} x_i$ for $j \in P$