Lesson 15. Multiperiod Models, Revisited

Problem 1. Priceler is planning the manufacture of its sedans and wagons for the next 12 months. For all $t \in \{1, \dots, 12\}$, the demand for sedans and wagons in month t are $d_{s,t}$ and $d_{w,t}$, respectively. Assume that the demand for both types of vehicle must be met exactly each month. Each sedan costs p_s to produce, and each wagon costs p_w to produce. Vehicles not sold in a given month can be held in inventory. To hold a vehicle in inventory from one month to the next costs h_s per sedan and h_w per wagon. During each month, at most K vehicles can be produced. At the beginning of month 1, v_s sedans and v_w wagons are available. Formulate a linear program that can be used to minimize Priceler's costs during the next 12 months.

Input parameters: T = set of months = $\{1, \dots, 12\}$
V = set of vehicles = $\{s, w\}$
$d_{i,t}$ = demand for vehicle i in month t for $i \in V$ and $t \in T$
p_i = cost of producing 1 vehicle i for $i \in V$
h_i = holding cost for 1 vehicle i for $i \in V$
v_i = initial inventory of vehicle i for $i \in V$
K = monthly limit on vehicles produced

DVs: $x_{i,t}$ = # vehicle i produced in month t for $i \in V$ and $t \in T$
$y_{i,t}$ = # vehicle i held at the end of month t for $i \in V$ and $t \in \underline{\underline{T \cup \{0\}}}$
union of 2 sets

$$\min \; p_s \sum_{t \in T} x_{s,t} + p_v \sum_{t \in T} x_{v,t} + h_s \sum_{t \in T} y_{s,t} + h_v \sum_{t \in T} y_{v,t} = \sum_{i \in V} p_i \sum_{t \in T} x_{i,t} + \sum_{i \in V} h_i \sum_{t \in T} x_{i,t}$$

$$\text{s.t.} \quad \left.\begin{aligned} & y_{i,t-1} + x_{i,t} = d_{i,t} + y_{i,t} \quad \text{for } i \in V,\ t \in T \\ & y_{i,0} = v_i \quad \text{for } i \in V \end{aligned}\right\} \text{(balance)}$$

$$\sum_{i \in V} x_{i,t} \le K \quad \text{for } t \in T \quad \text{(monthly limits)}$$

$$\left.\begin{aligned} & x_{i,t} \geq 0 \quad \text{for } i \in V,\ t \in T \\ & y_{i,t} \geq 0 \quad \text{for } i \in V,\ t \in T \cup \{0\} \end{aligned}\right\} \text{(nonnegativity)}$$

Problem 2. During the next 20 weeks, the Bellman Company would like to meet the demand for their line of advanced GPS navigation systems. In particular, the demand for GPS systems in week t is d_t for all $t \in \{1, \ldots, 20\}$. It takes 1 hour of labor to produce 1 GPS system. There are r_t regular-time labor hours available in week t, for all $t \in \{1, \ldots, 20\}$. In addition, each week, the company can require workers to put in up to M hours of overtime. Workers are only paid for the hours they work. A worker receives c_r per hour for regular-time work and c_o per hour for overtime work. GPS systems produced in a given week can be used to meet demand in that week , or put into a warehouse. Holding a GPS system in the warehouse from one week to the next costs h per GPS system. Formulate a linear program that minimizes the total cost incurred in meeting the demands of the next 20 weeks.

Input parameters: T = set of weeks = $\{1, \ldots, 20\}$

d_t = demand in week t for $t \in T$

r_t = regular-time hrs available in week t for $t \in T$

M = overtime hours available each week

c_r = regular time hourly cost

c_o = overtime hourly cost

h = holding cost per GPS

DVs: x_t = regular time GPS produced in week t for $t \in T$

y_t = overtime GPS produced in week t for $t \in T$

z_t = GPS held at end of month t for $t \in T \cup \{0\}$

$$\min \quad c_r \sum_{t \in T} x_t + c_o \sum_{t \in T} y_t + h \sum_{t \in T} z_t$$

$$\begin{aligned}
\text{s.t.} \quad & z_{t-1} + x_t + y_t = d_t + z_t && \text{for } t \in T \cup \{0\} && \text{(balance)} \\
& z_0 = 0 \\
& y_t \le M && \text{for } t \in T && \text{(overtime limit)} \\
& x_t \le r_t && \text{for } t \in T && \text{(regular time limit)} \\
& x_t \ge 0 && \text{for } t \in T \\
& y_t \ge 0 && \text{for } t \in T \\
& z_t \ge 0 && \text{for } t \in T \cup \{0\}
\end{aligned}$$