Lesson 26.

$$\max \quad 4x_1 + 3x_2 + 5x_3$$

Canonical form

$$\text{s.t.} \quad \begin{aligned} 2x_1 - x_2 + 4x_3 + s_1 \quad &= 18 \\ 4x_1 + 2x_2 + 5x_3 \quad + s_2 &= 10 \end{aligned}$$

$$x_1, x_2, x_3, s_1, s_2 \geq 0.$$

Let $\vec{x} = (x_1, x_2, x_3, s_1, s_2)$

$\vec{x}^0 = (0, 0, 0, 18, 10)$ $\quad \mathcal{B}^0 = \{s_1, s_2\}$.

$\underline{\vec{d}^{x_1}}$: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} d_{s_1} \\ d_{s_2} \end{pmatrix} = -\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\quad \Rightarrow \vec{d}^{x_1} = (1, 0, 0, -2, -4)$ $\quad \bar{c}_{x_1} = 4$

$\underline{\vec{d}^{x_2}}$: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} d_{s_1} \\ d_{s_2} \end{pmatrix} = -\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ $\quad \Rightarrow \vec{d}^{x_2} = (0, 1, 0, 1, -2)$ $\quad \bar{c}_{x_2} = 3$

$\underline{\vec{d}^{x_3}}$: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} d_{s_1} \\ d_{s_2} \end{pmatrix} = -\begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\quad \Rightarrow \vec{d}^{x_3} = (0, 0, 1, -4, -5)$. $\quad \boxed{\bar{c}_{x_3} = 5}$

Choose x_3 as entering.

$\underline{\text{MRT}}$: $\lambda_{max} = \min\left\{ \overset{s_1}{\frac{18}{4}}, \overset{s_2}{\underline{\frac{10}{5}}} \right\} = 2$ $\quad s_2$ is leaving.

$\Rightarrow \vec{x}^1 = \vec{x}^0 + \lambda_{max} \vec{d}^{x_3} = (0, 0, 0, 18, 10) + 2(0, 0, 1, -4, -5) = (0, 0, 2, 10, 0)$.

$\mathcal{B}^1 = \{x_3, s_1\}$.

$\vec{x}^1 = (0, 0, 2, 10, 0)$ $\quad \mathcal{B}^1 = \{x_3, s_1\}$.

$\underline{\vec{d}^{x_1}}$: $\begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} d_{x_3} \\ d_{s_1} \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ $\quad \Rightarrow \vec{d}^{x_1} = (1, 0, -\frac{4}{5}, \frac{6}{5}, 0)$ $\quad \bar{c}_{x_1} = 0$

$\underline{\vec{d}^{x_2}}$: $\begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} d_{x_3} \\ d_{s_1} \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ $\quad \Rightarrow \vec{d}^{x_2} = (0, 1, -\frac{2}{5}, \frac{13}{5}, 0)$ $\quad \boxed{\bar{c}_{x_2} = 1}$

$\underline{\vec{d}^{s_2}}$: $\begin{pmatrix} 4 & 1 \\ 5 & 0 \end{pmatrix}\begin{pmatrix} d_{x_3} \\ d_{s_1} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ $\quad \Rightarrow \vec{d}^{s_2} = (0, 0, -\frac{1}{5}, \frac{4}{5}, 1)$ $\quad \bar{c}_{s_2} = -1$

choose x_2 as entering.

$\underline{\text{MRT}}$: $\lambda_{max} = \min\left\{ \overset{x_3}{\frac{2}{2/5}} \right\} = 5$ $\quad x_3$ is leaving.

$\Rightarrow \vec{x}^2 = \vec{x}^1 + \lambda_{max} \vec{d}^{x_2} = (0, 0, 2, 10, 0) + 5(0, 1, -\frac{2}{5}, \frac{13}{5}, 0)$

$= (0, 5, 0, 23, 0)$.

$\mathcal{B}^2 = \{x_2, s_1\}$.

$$\vec{x}^2 = (0, 5, 0, 23, 0) \qquad \mathcal{B}^2 = \{x_2, s_1\}.$$

$$\underline{\vec{d}^{x_1}}: \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} d_{x_2} \\ d_{s_1} \end{pmatrix} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \qquad \underline{\vec{d}^{x_3}}: \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} d_{x_2} \\ d_{s_1} \end{pmatrix} = \begin{pmatrix} -4 \\ -5 \end{pmatrix} \qquad \underline{\vec{d}^{s_2}}: \begin{pmatrix} -1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} d_{x_2} \\ d_{s_1} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\Rightarrow \vec{d}^{x_1} = (1, -2, 0, -4, 0) \qquad \Rightarrow \vec{d}^{x_3} = \left(0, -\tfrac{5}{2}, 1, -\tfrac{13}{2}, 0\right) \qquad \Rightarrow \vec{d}^{s_2} = \left(0, -\tfrac{1}{2}, 0, -\tfrac{1}{2}, 1\right)$$

$$\bar{c}_{x_1} = 4 - 6 = -2 \qquad \bar{c}_{x_3} = -\frac{15}{2} + 5 = -\frac{5}{2} \qquad \bar{c}_{s_2} = -\frac{3}{2}.$$

No simplex directions are improving $\Rightarrow$ $\underline{\vec{x}^2 \text{ is optimal, w/ value } 15.}$

$\Rightarrow$ In the <u>original</u> LP, $\underline{(\overset{x_1}{0}, \overset{x_2}{5}, \overset{x_3}{0}) \text{ is an optimal solution, w/ opt. value } 15.}$