Lesson 31. Duality, Maximin Objectives

1 Practice taking duals!

Example 1. State the dual of the following linear programs.

a.
$$\begin{aligned} \text{minimize} \quad & 5x_1 + x_2 - 4x_3 \\ \text{subject to} \quad & x_1 + x_2 + x_3 + x_4 = 19 \\ & 4x_2 + 8x_4 \leq 55 \\ & x_1 + 6x_2 - x_3 \geq 7 \\ & x_2 \geq 0, x_3 \geq 0, x_4 \leq 0 \end{aligned}$$

b.
$$\begin{aligned} \text{maximize} \quad & 19y_1 + 4y_2 - 8z_2 \\ \text{subject to} \quad & 11y_1 + y_2 + z_1 = 15 \\ & z_1 + 5z_2 \leq 0 \\ & y_1 - y_2 + z_2 \geq 4 \\ & y_1 \geq 0, y_2 \geq 0 \end{aligned}$$

2 The minimum of a collection of functions

Example 2. Santa Claus is trying to decide how to give candy canes to three children: Ann, Bob, and Carol. Because Santa is a very busy person, he has decided to give the same number of candy canes to each child. Let x be the number of candy canes each child receives. Also, because Santa knows everything, he knows the happiness level of each child as a function of the number of candy canes he or she receives:

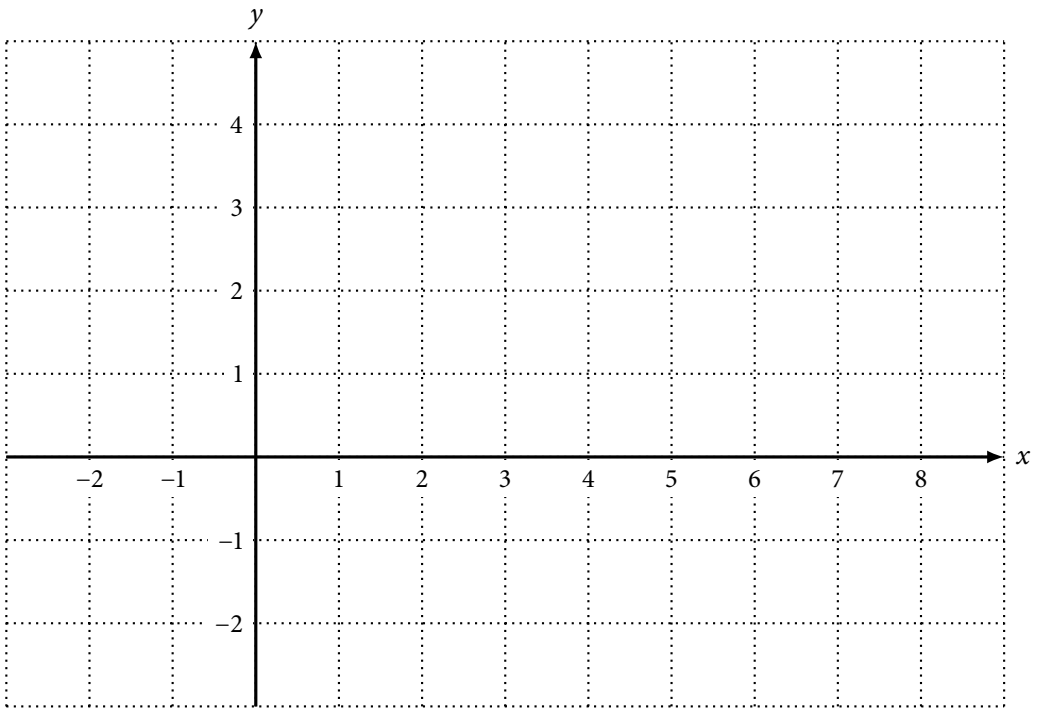
$$\text{Ann: } 1+2x \qquad \text{Bob: } 2+x \qquad \text{Carol: } 5-\frac{1}{2}x$$

Due to sequestration, Santa's budget limits him to give each child at most 6 candy canes. To be fair to all 3 children, he has decided that he wants to **maximize the minimum happiness level of all 3 children**. In other words, he is trying to maximize the worst-case happiness level.

Let $f(x)$ be the minimum happiness level of all 3 children when each child receives x candy canes:

What is $f(0)$? $f(1)$? $f(2)$?

Graph $f(x)$:



Santa's optimization problem is:

By looking at the graph of $f(x)$, give an optimal solution to Santa's optimization problem. What are Ann's, Bob's, and Carol's happiness levels at this solution?

Observation. The minimum of a collection of numbers is the largest value that is less than or equal to each number in the collection.

For example, consider $\min\{3, 8, -2, 6, 9\}$.

Using this observation, we can rewrite Santa's optimization problem as:

This looks familiar...

What if we maximized the sum of the happiness factors of all 3 children? What is the optimal solution? What is Ann's, Bob's, and Carol's happiness levels at this solution?

$\Longrightarrow$ **Maximizing the minimum results in more uniform performance than maximizing the sum**

3 Next time...

- Maximin objectives with two or more variables
- Minimax objectives