Lesson 35. Introduction to Networks and the Shortest Path Problem

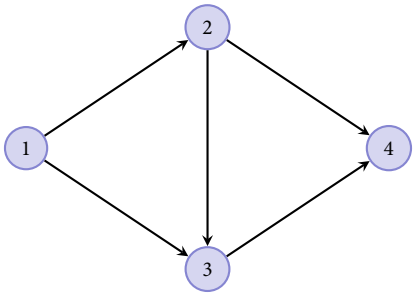
Today...

- What is the shortest way to get from Point A to Point B?

Graphs and networks

- **Graphs** model how various entities are connected
- A **directed graph** or **network** (V, A) consists of
  - set of **vertices** V (also known as **nodes**)
  - set of **arcs** A
    - arcs are directed from one vertex to another
    - arc from vertex i to vertex j is denoted by (i, j)

Example 1.



Networks are everywhere

- Physical networks
  - Road networks
  - Airline traffic networks
- Abstract networks
  - Organizational charts
  - Precedence relationships in projects
- Others?

Paths

- A **path** is a sequence of arcs connecting two specified vertices in a graph:
  - Each arc must have exactly one node in common with its predecessor in the sequence
  - Arcs must be passed in the forward direction
  - No vertex may be visited more than once

Example 2. Give an example of a path from vertex 1 to vertex 4 in the network in Example 1.

The shortest path problem

- Network (V, A)
- Each arc (i, j) in A has a **length** (or cost) c_{ij}
- Designate
  - one vertex in the network as the **source** s
  - another vertex in the network as the **sink** t
- What is the shortest path from s to t?

Example 3. Consider the network given in Example 1, with the following arc lengths: $c_{12} = 5$, $c_{13} = 4$, $c_{23} = 6$, $c_{24} = 3$, $c_{34} = 4$. Write an optimization model that finds the shortest path from vertex 1 to vertex 4.