SA305 – Linear Programming | Spring 2014
Asst. Prof. Nelson Uhan

Lesson 3. Graphical Solution of Optimization Models

0 Warm up

On the axes on page 2, draw the following equations, and label the points of intersection.

$$4C + 2V = 32 \qquad 4C + 6V = 48$$

1 Overview

- Last time, we formulated a linear program for Farmer Jones's problem:

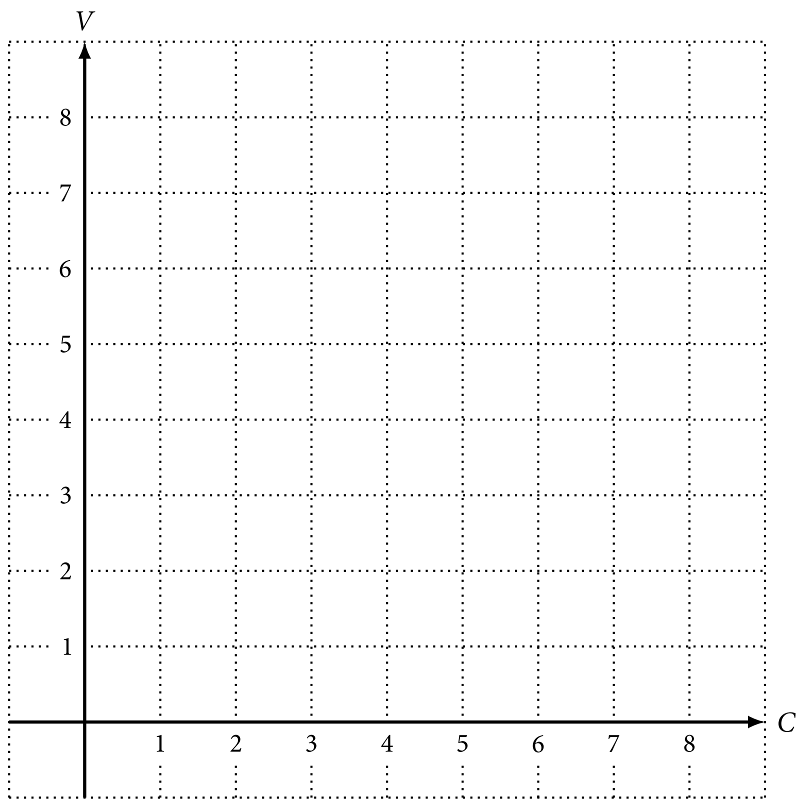
$$\begin{aligned} C &= \text{number of chocolate cakes to bake} \\ V &= \text{number of vanilla cakes to bake} \end{aligned}$$

$$\begin{aligned} \text{maximize} \quad & 3C + 4V \\ \text{subject to} \quad & 4C + 2V \leq 32 && (1) \\ & 4C + 6V \leq 48 && (2) \\ & C \geq 0 && (3) \\ & V \geq 0 && (4) \end{aligned}$$

- By trial-and-error, the best feasible solution we found was $C = 6$, $V = 4$ with value 34
- Today: let's find an optimal solution and the optimal value to Farmer Jones's model in a systematic way
- The **optimal value** of an optimization model is the value of an optimal solution

2 Solving Farmer Jones's model graphically

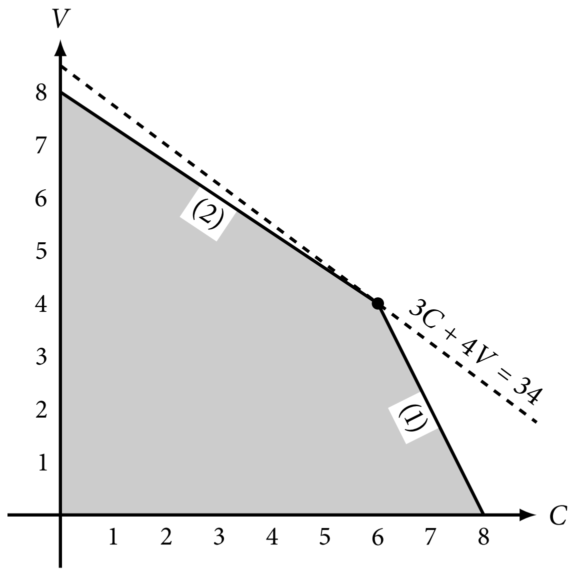
- We can graphically solve linear programs with 2 variables
- The feasible region – the collection of all feasible solutions – for Farmer Jones's optimization model:



- Any point in this shaded region represents a feasible solution
- How do we find the one with the highest value?
- $C = 6$, $V = 0$ is a feasible solution with value
- The set of values of C and V with the same value satisfies:
- Idea:
  - Draw lines of the form $3C + 4V = k$ for different values of k
  - Find the largest value of k such that the line $3C + 4V = k$ intersects the feasible region
- These lines are called **contour plots**
  - Lines through points having <u>equal objective function value</u>

3 Sensitivity analysis

- For what profit margins on vanilla cakes will the current optimal solution remain optimal?



- Key observation:

- Slope of (1) = [ ], slope of (2) = [ ]

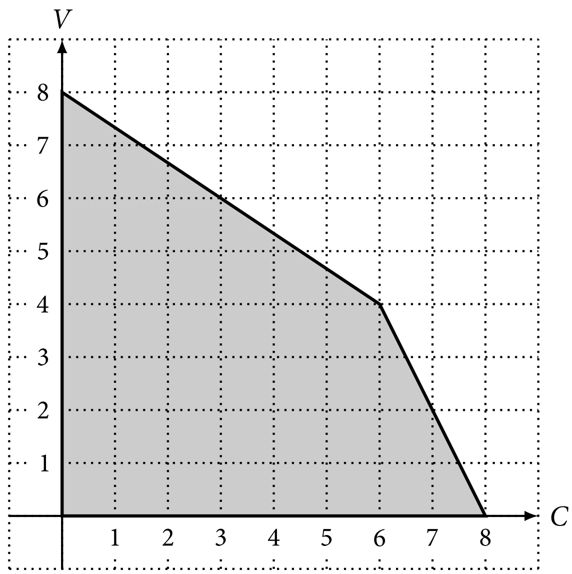
- Let a be the new profit margin on vanilla cakes

  ⇒ objective function is [ ], slope of contour plots = [ ]

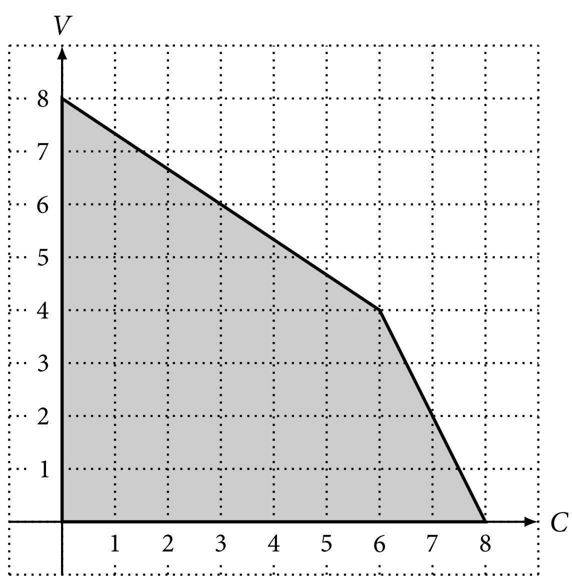
⇒ If [ ], then the current optimal solution remains optimal

4 Outcomes of optimization models

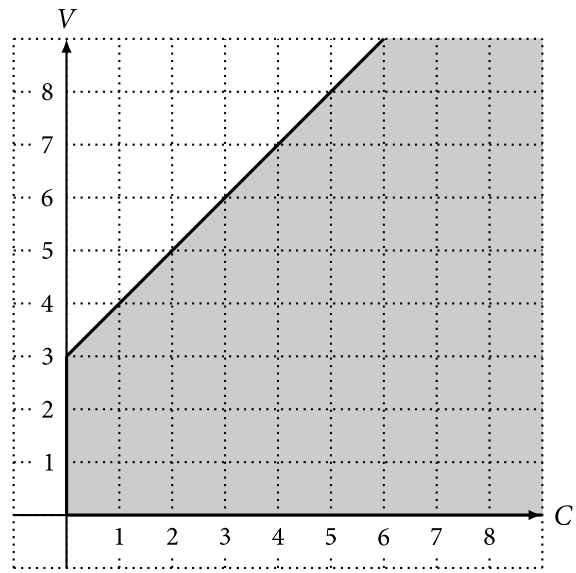
- An optimization model may:
  1. have a **unique optimal solution**
     - e.g. the original Farmer Jones model
  2. have **multiple optimal solutions**
     - e.g. What if the profit margin on chocolate and vanilla cakes is \$2 and \$3, respectively, instead?
     - Farmer Jones's objective function is then



  3. be **infeasible**: no choice of decision variables satisfies all constraints
     - e.g. What if the demands of Farmer Jones's neighbors dictate that he needs to bake at least 9 chocolate cakes?
     - Then we need to add the constraint



4. be **unbounded**: for any feasible solution, there exists another feasible solution with a better value
   - e.g. What if the circumstances have changed so that the feasible region of Farmer Jones's model actually looks like this:



5 Next time...

- More linear programming models
- Introduction to GMPL (bring your laptops)