SA305 – Linear Programming
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Lesson 5. Work Scheduling Models

Example 1. Postal employees in Simplexville work for 5 consecutive days, followed by 2 days off, repeated weekly. Below are the minimum number of employees needed for each day of the week:

Day	Employees needed
Monday (1)	7
Tuesday (2)	8
Wednesday (3)	7
Thursday (4)	6
Friday (5)	6
Saturday (6)	4
Sunday (7)	5

Write a linear program that determines the minimum total number of employees needed. You may assume that fractional solutions are acceptable.

DVs: x_1 = # employees working on day 1
x_2 = " " " " 2
⋮
x_7 = # employees working on day 7

y_1 = # employees who start on day 1 + end on day 5
y_2 = # employees who start on day 2 + end on day 6
⋮
y_7 = # employees who $\underbrace{\text{start on day 7 + end on day 4}}_{\text{Sun, Mon, Tue, Wed, Thu}}$

Problem: How can we express "5 consecutive days followed by 2 days off"?

$$
\begin{aligned}
\min \quad & y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7 \\
\text{s.t.} \quad & \underbrace{y_1 + y_4 + y_5 + y_6 + y_7}_{\text{which employees have a shift that includes Monday?}} \geq 7 \quad \text{(Mon)} \\
& y_1 + y_2 + y_5 + y_6 + y_7 \geq 8 \quad \text{(Tue)} \\
& y_1 + y_2 + y_3 + y_6 + y_7 \geq 7 \quad \text{(Wed)} \\
& y_1 + y_2 + y_3 + y_4 + y_7 \geq 6 \quad \text{(Thu)} \\
& y_1 + y_2 + y_3 + y_4 + y_5 \geq 6 \quad \text{(Fri)} \\
& y_2 + y_3 + y_4 + y_5 + y_6 \geq 4 \quad \text{(Sat)} \\
& y_3 + y_4 + y_5 + y_6 + y_7 \geq 5 \quad \text{(Sun)} \\
& y_1 \geq 0,\ y_2 \geq 0,\ \ldots,\ y_7 \geq 0 \quad \text{(nonnegativity)}
\end{aligned}
$$

Example 2. At Melanie's Kitchen, tables are set and cleared by runners working 4-hour shifts that start on the hour, from 5am to 10am. For example, the shift that starts at 9am ends at 1pm. Melanie's pays \$7 per hour for the shifts that start at 5am, 6am, and 7am, and \$6 per hour for the shifts that start at 8am, 9am, and 10am. Past experience indicates that the following number of runners are needed at each hour of operation:

Hour	Number of runners required
5am-6am	2
6am-7am	3
7am-8am	5
8am-9am	5
9am-10am	4
10am-11am	3
11am-12pm	2
12pm-1pm	5
1pm-2pm	6

Formulate a linear program that determines a cost-minimizing staffing plan. You may assume that fractional solutions are acceptable.

DVs: x_5 = # runners starting at 5am and ending at 9am

x_6 = " "

$\vdots$

x_{10} = # runners starting at 10am and ending at 2pm

$$
\begin{aligned}
\text{min} \quad & 28(x_5 + x_6 + x_7) + 24(x_8 + x_9 + x_{10}) \\
\text{s.t.} \quad & x_5 \geq 2 \quad (5\text{-}6) \\
& x_5 + x_6 \geq 3 \quad (6\text{-}7) \\
& x_5 + x_6 + x_7 \geq 5 \quad (7\text{-}8) \\
& x_5 + x_6 + x_7 + x_8 \geq 5 \quad (8\text{-}9) \\
& x_6 + x_7 + x_8 + x_9 \geq 4 \quad (9\text{-}10) \\
& x_7 + x_8 + x_9 + x_{10} \geq 3 \quad (10\text{-}11) \\
& x_8 + x_9 + x_{10} \geq 2 \quad (11\text{-}12) \\
& x_9 + x_{10} \geq 5 \quad (12\text{-}1) \\
& x_{10} \geq 6 \quad (1\text{-}2) \\
& x_5 \geq 0,\ x_6 \geq 0,\ \ldots,\ x_{10} \geq 0 \quad (\text{nonnegativity})
\end{aligned}
$$