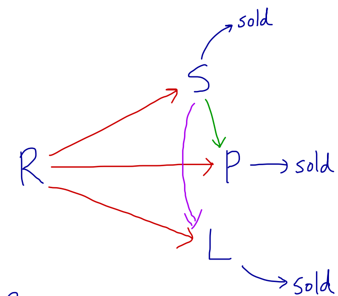
Lesson 14. Production Process Models, Revisited

Example 1. Yobro Co. produces three types of high-end organic, bio-diverse, fair-trade, non-harmful-to-animals household cleaners: standard, pine, and lemon. Each gallon of raw soap produces a_s gallons of standard, a_p gallons of pine, and a_ℓ gallons of lemon. Each gallon of standard can be converted directly into b_{sp} gallons of pine at a cost of c_{sp} per gallon. Separately, each gallon of standard can also be converted into $b_{s\ell}$ gallons of lemon at a cost of $c_{s\ell}$ per gallon. Raw soap costs c_r per gallon. Standard, pine, and lemon sell for v_s, v_p, and v_ℓ per gallon, respectively. Suppose that Yobro wants to satisfy demand for d_s gallons of standard, d_p of pine, and d_ℓ gallons of lemon.

a. Write a linear program that determines the number of gallons of each type of cleaner Yobro should make in order to maximize profit. Make sure to
   - define the input parameters,
   - define the decision variables, and
   - briefly explain the objective function and constraints that you write.



Input parameters

C = set of cleaners = $\{s, p, \ell\}$

a_i = gal. of cleaner i produced from 1 gal. raw soap, for $i \in C$

b_{si} = gal. of cleaner i produced from 1 gal. standard, for $i \in \{p, \ell\}$

c_{si} = cost of converting 1 gal. standard to cleaner i, for $i \in \{p, \ell\}$

c_r = cost of 1 gal. raw soap

v_i = revenue from 1 gal. cleaner i, for $i \in C$

d_i = demand (in gal.) for cleaner i, for $i \in C$

DVs:

r = gal. raw soap used

x_{si} = gal. standard converted to cleaner i, for $i \in \{p, \ell\}$

x_i = gal. cleaner i sold, for $i \in C$

$$
\begin{aligned}
\max \quad & \sum_{i \in C} v_i x_i - c_r r - \sum_{i \in \{p,\ell\}} c_{si} x_{si} && \text{(total profit)} \\
\text{s.t.} \quad & \left.\begin{aligned} & a_s r = x_s + x_{sp} + x_{s\ell} && \text{(std. balance)} \\ & a_p r + b_{sp} x_{sp} = x_p && \text{(pine balance)} \\ & a_\ell r + b_{s\ell} x_{s\ell} = x_\ell && \text{(lemon balance)} \end{aligned}\right\} (*) \\
& x_i \geq d_i \quad \text{for } i \in C && \text{(demand)} \\
& r \geq 0, \; x_{si} \geq 0 \text{ for } i \in \{p,\ell\}, \; x_i \geq 0 \text{ for } i \in C && \text{(nonnegativity)}
\end{aligned}
$$

b. YoBro just tweeted that they have created an additional process that converts standard to pine and lemon simultaneously. With this process, each gallon of standard converts to f_{sp} gallons of pine and $f_{s\ell}$ gallons of lemon at a cost of $c_{sp\ell}$ per gallon. How do you change the linear program you just wrote to account for this new process?

Add input parameters: f_{sp} = gal. pine produced from 1 gal. standard w/new process
$f_{s\ell}$ = gal. lemon produced from 1 gal. standard w/new process
$c_{sp\ell}$ = cost of converting 1 gal. standard w/new process

Add DV: $x_{sp\ell}$ = gal. standard converted to pine + lemon w/new process

Add to obj. fn.: $-c_{sp\ell}\, x_{sp\ell}$

Change balance constraints (*):

$$a_s r = x_s + x_{sp} + x_{s\ell} + x_{sp\ell} \quad \text{(std. bal.)}$$
$$a_p r + b_{sp} x_{sp} + f_{sp} x_{sp\ell} = x_p \quad \text{(pine bal.)}$$
$$a_\ell r + b_{s\ell} x_{s\ell} + f_{s\ell} x_{sp\ell} = x_\ell \quad \text{(lemon bal.)}$$

Bonus: write a for statement that captures these 2 constraints