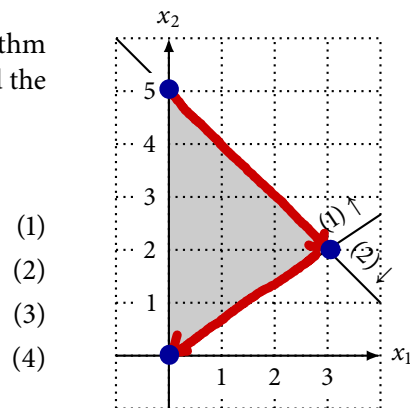


Lesson 19 / Quiz 6. Improving Search Review

Instructions. You have 40 minutes to complete this quiz. You may use your notes, homework, and textbook during this quiz. You must work in teams of 2 or 3.

This problem will walk you through the improving search algorithm on a small linear program. Consider the linear program below and the graph of its feasible region to the right.

$$\begin{aligned} &\text{minimize} && x_2 \\ &\text{subject to} && 2x_1 - 3x_2 \leq 0 \quad (1) \\ & && x_1 + x_2 \leq 5 \quad (2) \\ & && x_1 \geq 0 \quad (3) \\ & && x_2 \geq 0 \quad (4) \end{aligned}$$



a. Let $\mathbf{x}^0 = (0, 5)$. Which constraints are active at \mathbf{x}^0 ?

$$(2), (3)$$

b. Write the constraints that a feasible direction $\mathbf{d} = (d_1, d_2)$ at \mathbf{x}^0 must satisfy.

$$d_1 + d_2 \leq 0 \quad \leftarrow \text{from constraint (2)}$$

$$d_1 \geq 0 \quad \leftarrow \text{from constraint (3)}$$

c. Using your answer from part b, is $\mathbf{d} = (1, -1)$ feasible at \mathbf{x}^0 ?

Yes, since it satisfies both constraints in part b.

d. Call the objective function f ; that is, $f(\mathbf{x}) = x_2$. Using $\mathbf{d} = (1, -1)$, compute $\nabla f(\mathbf{x}^0)^T \mathbf{d}$. Is \mathbf{d} improving at \mathbf{x}^0 ? Why?

$$\nabla f(\bar{\mathbf{x}}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \nabla f(\bar{\mathbf{x}})^T \bar{\mathbf{d}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -1 < 0 \quad \leftarrow \text{Indicates that } f \text{ decreases when moving in the } \bar{\mathbf{d}} \text{ direction}$$

$$\Rightarrow \text{Since this is a minimizing LP, } \bar{\mathbf{d}} \text{ is improving at } \bar{\mathbf{x}}^0$$

e. Using $\lambda = 3$ and $\mathbf{d} = (1, -1)$, compute $\mathbf{x}^1 = \mathbf{x}^0 + \lambda \mathbf{d}$.

$$\bar{\mathbf{x}}^1 = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + 3 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

- f. Trace the trajectory $\mathbf{x}^0 \rightarrow \mathbf{x}^1$ on the graph above. Note that if $\lambda > 3$, $\mathbf{x}^0 + \lambda \mathbf{d}$ is outside the feasible region.
- g. Now, which constraints are active at \mathbf{x}^1 , the solution you computed in part e?

$$(1), (2)$$

- h. Write the constraints that a feasible direction $\mathbf{d} = (d_1, d_2)$ at \mathbf{x}^1 must satisfy.

$$\begin{aligned} 2d_1 - 3d_2 &\leq 0 \\ d_1 + d_2 &\leq 0 \end{aligned}$$

- i. Using your answer from h, is $\mathbf{d} = (-3, -2)$ feasible at \mathbf{x}^1 ?

Yes, since it satisfies both constraints in part h.

- j. Using $\mathbf{d} = (-3, -2)$, compute $\nabla f(\mathbf{x}^1)^T \mathbf{d}$. Is \mathbf{d} improving at \mathbf{x}^1 ? Why?

$$\begin{aligned} \nabla f(\bar{\mathbf{x}}^1) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \nabla f(\bar{\mathbf{x}}^1)^T \vec{\mathbf{d}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \begin{pmatrix} -3 \\ -2 \end{pmatrix} = -2 < 0 \\ \Rightarrow &\text{Since this is a minimizing LP, } \vec{\mathbf{d}} \text{ is improving at } \bar{\mathbf{x}}^1. \end{aligned}$$

- k. Using $\lambda = 1$ and $\mathbf{d} = (-3, -2)$, compute $\mathbf{x}^2 = \mathbf{x}^1 + \lambda \mathbf{d}$.

$$\vec{\mathbf{x}}^2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + 1 \begin{pmatrix} -3 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- l. Trace the trajectory $\mathbf{x}^1 \rightarrow \mathbf{x}^2$ on the graph above. Note that if $\lambda > 1$, $\mathbf{x}^1 + \lambda \mathbf{d}$ is outside the feasible region.
- m. In words, are there any simultaneously improving and feasible directions at \mathbf{x}^2 ? Why or why not?

No. Any feasible direction must be pointing "upwards," and since we want to minimize x_2 , any improving direction must be pointing "downwards."

Preview: the simplex method

- The simplex method is an improving search algorithm for linear programs
- It moves between "corner points" of the feasible region
- This implies specific ways of computing improving and feasible directions and step sizes
- Next time: geometry and algebra of "corner points"