SA305 – Linear Programming
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Spring 2014

Lesson 33. Maximin and Minimax Objectives, cont.

Example 1. Captain Hook, Captain Sparrow, and Captain Crunch have "found" some treasure. They can't agree on how to split their newly found riches, and so they have asked you for help. To start, you ask them to assign "value points" to all of the items under consideration, so that the points add up to 100. Their valuations are shown below:

	Item	Captain Hook	Captain Sparrow	Captain Crunch
1	Gold coins	40	30	5
2	Jewels	10	25	5
3	Spices	45	40	10
4	Cereal	5	5	80

Assume that the items can be divided fractionally. For example, Captain Hook can receive 50% of the gold coins, Captain Sparrow can receive 30%, and Captain Crunch can receive the remaining 20%. In this case, Captain Hook gets $0.5 \times 40 = 20$ value points, Captain Sparrow gets $0.3 \times 30 = 9$ value points, and Captain Crunch gets $0.2 \times 5 = 1$ value point.

Formulate a linear program that determines how to allocate the treasure between Captain Hook, Captain Sparrow and Captain Crunch, in a way that maximizes the minimum total value points of any captain.

Decision variables: $x_i = \%$ of item i assigned to Hook for $i \in \{1,2,3,4\}$
$y_i = \%$ of item i assigned to Sparrow for $i \in \{1,2,3,4\}$
$z_i = \%$ of item i assigned to Crunch for $i \in \{1,2,3,4\}$

$$\begin{aligned}
\max \quad & \min\{40x_1 + 10x_2 + 45x_3 + 5x_4,\ 30y_1 + 25y_2 + 40y_3 + 5y_4,\ 5z_1 + 5z_2 + 10z_3 + 80z_4\} \\
\text{s.t.} \quad & x_i + y_i + z_i = 1 \quad \text{for } i \in \{1,2,3,4\} \\
& x_i \geq 0,\ y_i \geq 0,\ z_i \geq 0 \quad \text{for } i \in \{1,2,3,4\}
\end{aligned}$$

Reformulate as LP:

$$\begin{aligned}
\max \quad & w \\
\text{s.t.} \quad & w \leq 40x_1 + 10x_2 + 45x_3 + 5x_4 \\
& w \leq 30y_1 + 25y_2 + 40y_3 + 5y_4 \\
& w \leq 5z_1 + 5z_2 + 10z_3 + 80z_4 \\
& x_i + y_i + z_i = 1 \quad \text{for } i \in \{1,2,3,4\} \\
& x_i \geq 0,\ y_i \geq 0,\ z_i \geq 0 \quad \text{for } i \in \{1,2,3,4\}
\end{aligned}$$

Using symbolic input parameters:

<u>Input parameters:</u>

P = set of pirates

I = set of items

a_{ij} = value of item i for pirate j for $i \in I$, $j \in P$

<u>Decision variables:</u> x_{ij} = % of item i assigned to pirate j for $i \in I$, $j \in P$

$$\max \quad \min \left\{ \sum_{i \in I} a_{ij} x_{ij} \; : \; j \in P \right\}$$

$$\text{s.t.} \quad \sum_{j \in P} x_{ij} = 1 \quad \text{for } i \in I$$

$$x_{ij} \geq 0 \quad \text{for } i \in I,\ j \in P$$

if $P = \{1, \dots, m\}$, this is the same as saying:

$$\min \left\{ \sum_{i \in I} a_{i1} x_{i1}, \sum_{i \in I} a_{i2} x_{i2}, \dots, \sum_{i \in I} a_{im} x_{im} \right\}$$

Reformulate as an LP:

$$\max \quad w$$

$$\text{s.t.} \quad w \leq \sum_{i \in I} a_{ij} x_{ij} \quad \text{for } j \in P$$

$$\sum_{j \in P} x_{ij} = 1 \quad \text{for } i \in I$$

$$x_{ij} \geq 0 \quad \text{for } i \in I,\ j \in P$$