

## Lesson 10. Sets, Summations, For Statements

### 1 Sets

- A **set** is a collections of elements/objects, e.g.

$$S = \{1, 2, 3, 4, 5\} \quad \text{Fruits} = \{\text{Apple, Orange, Pear}\} \quad (1)$$

- “in” symbol:

$$i \in N \Leftrightarrow \text{“element } i \text{ is in the set } N\text{”}$$

- For example:

### 2 Summations

- Summation symbol over sets:

$$\sum_{i \in N} \Leftrightarrow \text{“sum over all elements of } N\text{”}$$

- For example:

- Common shorthand: if  $N = \{1, 2, \dots, n\}$ , then

$$\sum_{i \in N} \text{ is the same as } \sum_{i \in \{1, 2, \dots, n\}} \text{ as well as } \sum_{i=1}^n$$

**Problem 1.** Let the sets  $S$  and  $\text{Fruits}$  be defined as above in (1). Write each of the following as compactly as possible using summation notation:

- $x_{\text{Apple}} + x_{\text{Orange}} + x_{\text{Pear}}$
- $1y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5$

### 3 For statements

- “for” statements over sets:

$$\text{for } i \in N \iff \text{“repeat for each element of } N\text{”}$$

- For example:

$$c_j x_1 + d_j x_2 \leq b_j \quad \text{for } j \in \text{Fruits} \iff$$

- Common shorthand: if  $N = \{1, 2, \dots, n\}$ , then

“for  $i \in N$ ” is the same as “for  $i \in \{1, 2, \dots, n\}$ ” as well as “for  $i = 1, 2, \dots, n$ ”

- Sometimes we say “for all  $i \in N$ ” instead of “for  $i \in N$ ”

### 4 Multiple indices

- Sometimes it may be useful to use decision variables with multiple indices

- Example:

- Set of hat types:  $H = \{A, B, C\}$
- Set of factories:  $F = \{1, 2\}$
- Each hat type can be produced at each factory
- Define decision variables:

$$x_{i,j} = \text{number of type } i \text{ hats produced at factory } j \quad \text{for } i \in H \text{ and } j \in F \quad (2)$$

- What decision variables have we just defined? How many are there?

**Problem 2.** Using the decision variables defined in (2), write expressions for

- Total number of type C hats produced
- Total number of hats produced at facility 2

Use summation notation if possible.

- Suppose

$$c_{i,j} = \text{cost of producing one type } i \text{ hat at factory } j \quad \text{for } i \in H \text{ and } j \in F$$

- If we produce  $x_{i,j}$  hats of type  $i$  at factory  $j$  (for  $i \in H$  and  $j \in F$ ), then the total cost is

**Problem 3.** Let  $M = \{1, 2, 3\}$  and  $N = \{1, 2, 3, 4\}$ . Write the following as compactly as possible using summation notation and “for” statements.

Let  $y_1$  = amount of product 1 produced  
 $y_2$  = amount of product 2 produced  
 $y_3$  = amount of product 3 produced  
 $y_4$  = amount of product 4 produced

$$a_{1,1}y_1 + a_{1,2}y_2 + a_{1,3}y_3 + a_{1,4}y_4 = b_1$$

$$a_{2,1}y_1 + a_{2,2}y_2 + a_{2,3}y_3 + a_{2,4}y_4 = b_2$$

$$a_{3,1}y_1 + a_{3,2}y_2 + a_{3,3}y_3 + a_{3,4}y_4 = b_3$$