# Lesson 14. Production Process Models, Revisited

**Example 1.** Yobro Co. produces three types of high-end organic, bio-diverse, fair-trade, non-harmful-to-animals household cleaners: standard, pine, and lemon. Each gallon of raw soap produces $a_s$ gallons of standard, $a_p$ gallons of pine, and $a_\ell$ gallons of lemon. Each gallon of standard can be converted directly into $b_{sp}$ gallons of pine at a cost of $c_{sp}$ per gallon. Separately, each gallon of standard can also be converted into $b_{s\ell}$ gallons of lemon at a cost of $c_{s\ell}$ per gallon. Raw soap costs $c_r$ per gallon. Standard, pine, and lemon sell for $v_s$, $v_p$, and $v_\ell$ per gallon, respectively. Suppose that Yobro wants to satisfy demand for $d_s$ gallons of standard, $d_p$ of pine, and $d_\ell$ gallons of lemon.

a. Write a linear program that determines the number of gallons of each type of cleaner Yobro should make in order to maximize profit. Make sure to
   - define the input parameters,
   - define the decision variables, and
   - briefly explain the objective function and constraints that you write.

Input parameters:

$a_i$ = gal. of cleaner $i$ from 1 gal. raw soap for $i \in \{s, p, \ell\}$
$b_{si}$ = gal. of cleaner $i$ from 1 gal. standard for $i \in \{p, \ell\}$
$c_r$ = cost of 1 gal. raw soap
$c_{si}$ = cost of 1 gal. standard $\rightarrow$ cleaner $i$ for $i \in \{p, \ell\}$
$v_i$ = revenue of 1 gal. cleaner $i$ for $i \in \{s, p, \ell\}$
$d_i$ = demand for cleaner $i$ for $i \in \{s, p, \ell\}$

DVs: $x_r$ = gal. raw soap purchased
$x_{si}$ = gal. standard $\rightarrow$ cleaner $i$ for $i \in \{p, \ell\}$
$y_i$ = gal. cleaner $i$ sold for $i \in \{s, p, \ell\}$

$$\max \sum_{i \in \{s,p,\ell\}} v_i y_i - c_r x_r - \sum_{i \in \{p,\ell\}} c_{si} x_{si} \quad \text{(total profit)}$$

$$\text{s.t. } a_s x_r = x_{sp} + x_{s\ell} + y_s \quad \text{(standard balance)}$$

$$a_i x_r + b_{si} x_{si} = y_p \quad \text{for } i \in \{p, \ell\} \quad \text{(pine, lemon balance)}$$

$$y_i \geq d_i \quad \text{for } i \in \{s, p, \ell\}$$

$$\left.\begin{aligned} x_r &\geq 0 \\ x_{si} &\geq 0 \quad \text{for } i \in \{p, \ell\} \\ y_i &\geq 0 \quad \text{for } i \in \{s, p, \ell\} \end{aligned}\right\} \quad \text{(nonnegativity)}$$

b. YoBro just tweeted that they have created an additional process that converts standard to pine and lemon simultaneously. With this process, each gallon of standard converts to $f_{sp}$ gallons of pine and $f_{s\ell}$ gallons of lemon at a cost of $c_{sp\ell}$ per gallon. How do you change the linear program you just wrote to account for this new process?

Add input parameters:

$f_{si}$ = gal. cleaner $i$ from standard using simultaneous process for $i \in \{p, \ell\}$

$C_{sp\ell}$ = cost of 1 gal. standard $\rightarrow$ pine + lemon simultaneously

Add DV: $x_{sp\ell}$ = gal. standard $\rightarrow$ pine + lemon simultaneously

Add to objective function: $-C_{sp\ell}\, x_{sp\ell}$

Change constraints:

$$a_s x_r = x_{sp} + x_{s\ell} + y_s + x_{sp\ell}$$

$$f_{si}\, x_{sp\ell} + a_i x_r + b_{si}\, x_{si} = y_p \quad \text{for } i \in \{p, \ell\}$$

$$y_i \geq d_i \quad \text{for } i \in \{s, p, \ell\}$$

$$x_r \geq 0$$

$$x_{si} \geq 0 \quad \text{for } i \in \{p, \ell\}$$

$$y_i \geq 0 \quad \text{for } i \in \{s, p, \ell\}$$

$$x_{sp\ell} \geq 0$$