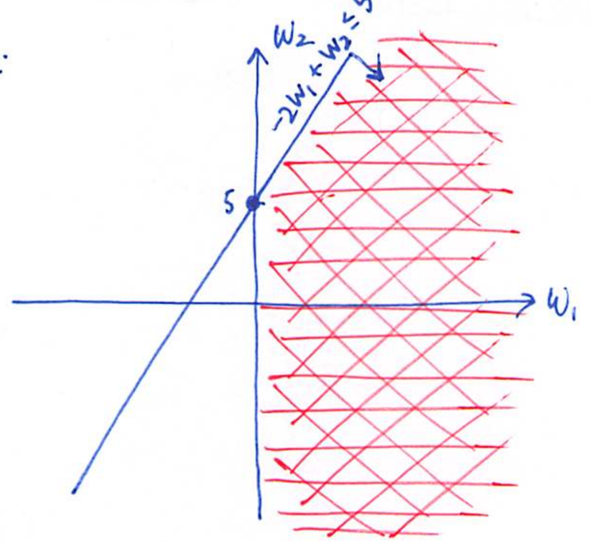
P1.



Graph of feasible region

a. minimize $3w_1 - w_2$

$\Rightarrow$ opt. soln. is $(0,5)$

b. minimize w_1

$\Rightarrow$ opt. soln. is the intersection of the w_2-axis and the feasible region

c. minimize $-w_1$

$\Rightarrow$ model is unbounded (obj. fn. contours with better obj. fn. value "move to the right" without limit.)

P2. a. Current basis $= \{x_1, x_3\} \Rightarrow x_2 = x_4 = 0$ at current BFS:

$$\left.\begin{aligned} -x_1 + 4x_3 &= 13 \\ 2x_1 &= 2 \end{aligned}\right\} \Rightarrow x_1 = 1,\ x_3 = \frac{14}{4} = \frac{7}{2}$$

$\Rightarrow$ current BFS $= (1, 0, \frac{7}{2}, 0)$.

b. $\vec{d}^{x_2} = (d_{x_1}, 1, d_{x_3}, 0)$

$$-d_{x_1} + 4d_{x_3} = -1$$
$$2d_{x_1} = -6$$

$\Rightarrow d_{x_1} = -3,\ d_{x_3} = -1$

$\Rightarrow \vec{d}^{x_2} = (-3, 1, -1, 0)$

$\vec{d}^{x_4} = (d_{x_1}, 0, d_{x_3}, 1)$

$$-d_{x_1} + 4d_{x_3} = -21$$
$$2d_{x_1} = 2$$

$\Rightarrow d_{x_1} = 1,\ d_{x_3} = -5$

$\Rightarrow \vec{d}^{x_4} = (1, 0, -5, 1)$.

c. $\bar{c}_{x_2} = -30 + 1 = -29$

$\Rightarrow \vec{d}^{x_2}$ not improving

$\bar{c}_{x_4} = 10$

$\Rightarrow \vec{d}^{x_4}$ improving.

d. Choose $\vec{d}^{x_4}$: x_4 is the entering variable

$\lambda_{max} = \min\left\{\frac{7/2}{5}\right\} = \frac{7}{10}$ $\quad x_3$ is the leaving variable.

$\Rightarrow$ new BFS $= (1, 0, \frac{7}{2}, 0) + \frac{7}{10}(1, 0, -5, 1) = (\frac{17}{10}, 0, 0, \frac{7}{10})$

new basis $= \{x_1, x_4\}$.

P3. a. min $5y_1 + 8y_2$

s.t. $y_1 + 2y_2 \geq 3$ (1)

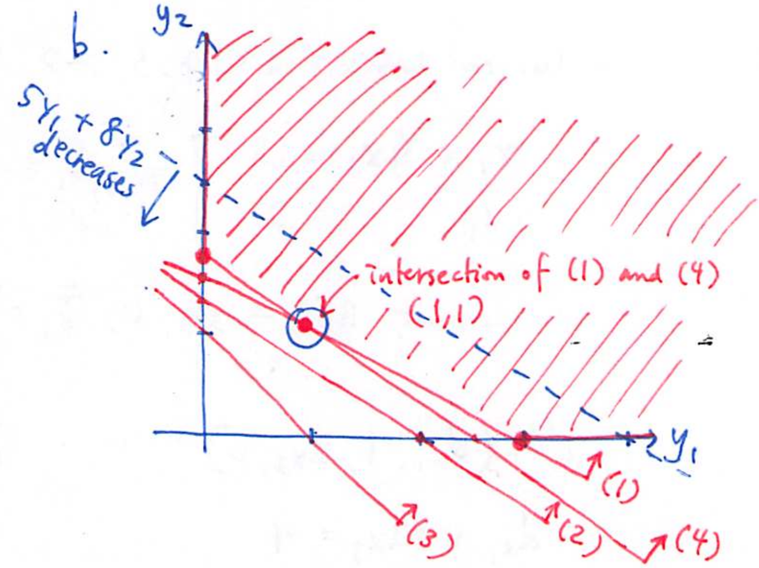
$2y_1 + 3y_2 \geq 4$ (2)

$y_1 + y_2 \geq 1$ (3)

$2y_1 + 3y_2 \geq 5$ (4)

$y_1, y_2 \geq 0$.

b.



optimal soln: $y_1^* = 1$, $y_2^* = 1$.

c. Dual complementary slackness: dual constraint not active

$\Rightarrow$ corresponding primal variable $= 0$.

From part b, it is clear that constraints (2) and (3) are not active at $(1,1)$. $\Rightarrow$ In an optimal solution to [P], we must have $x_2 = x_3 = 0$.

d. modified [P]:

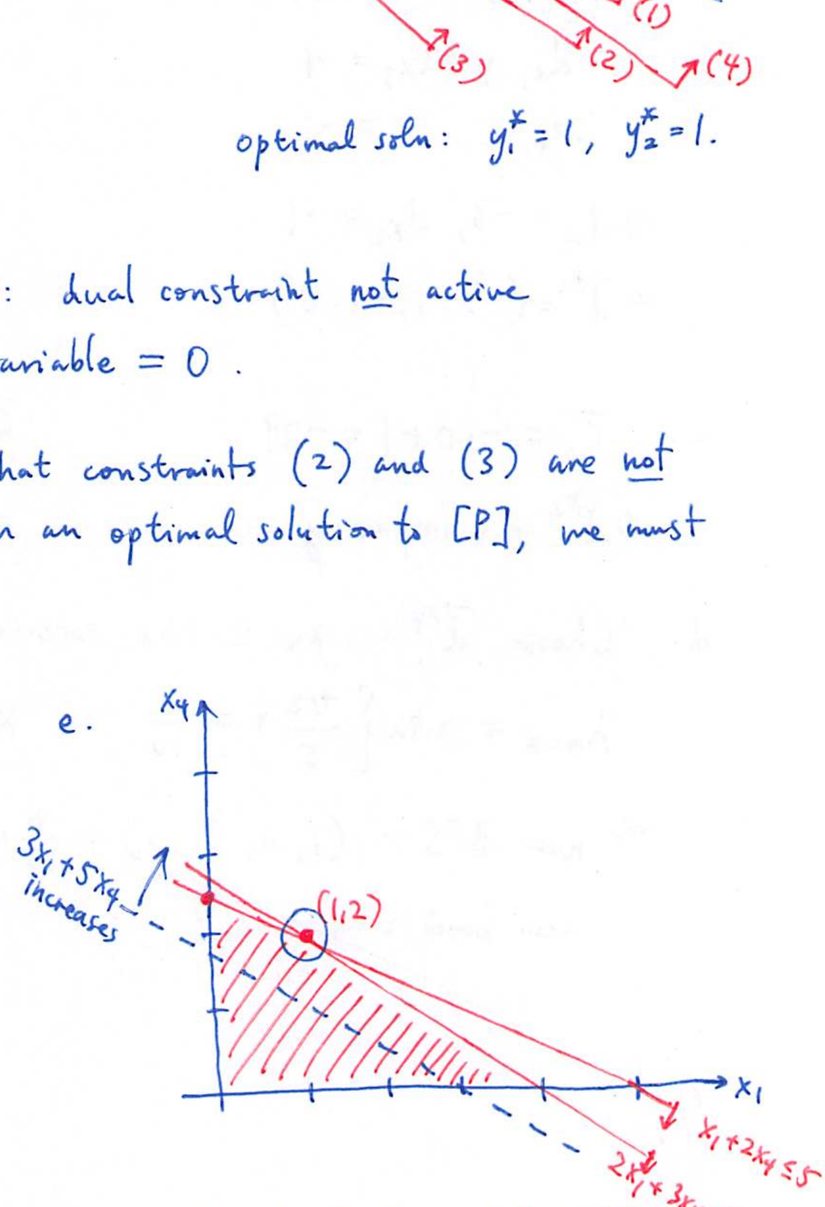
max $3x_1 + 5x_4$

s.t. $x_1 + 2x_4 \leq 5$

$2x_1 + 3x_4 \leq 8$

$x_1, x_4 \geq 0$

e.



optimal soln to modified [P]:

$x_1^* = 1$, $x_4^* = 2$

optimal soln. to [P]: $(1, 0, 0, 2)$

P4. Symbolic input parameters:

c = cost of crude oil per 1000 barrels

p_a = price of unprocessed aviation fuel per 1000 barrels

p_h = price of unprocessed heating oil per 1000 barrels

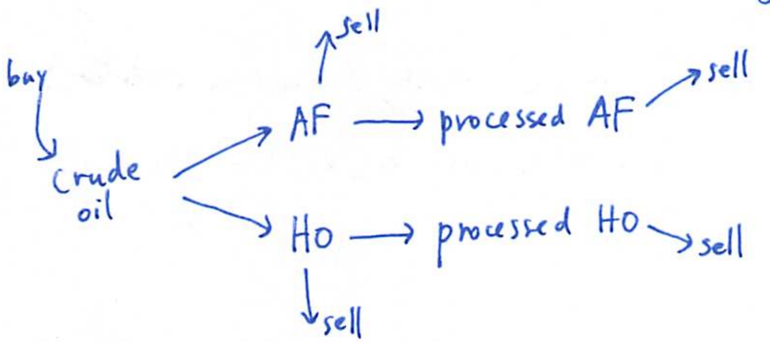
q_a = price of processed aviation fuel per 1000 barrels

q_h = price of processed heating oil per 1000 barrels

t_a = time to process aviation fuel per 1000 barrels

t_h = time to process heating oil per 1000 barrels

B = available crude oil, in 1000s.



Decision variables: (in 1000 barrels)

z = amt. of crude oil to buy

x_a = amt. of unprocessed AF to sell

x_h = amt. of unprocessed Ho to sell

y_a = amt. of processed AF to sell

y_h = amt. of processed Ho to sell.

Model:

$$\max \quad p_a x_a + p_h x_h + q_a y_a + q_h y_h - cz \qquad \text{(total profit)}$$

$$\text{s.t.} \quad \frac{3}{4} z = x_a + y_a \qquad \text{(crude oil} \rightarrow \text{AF)}$$

$$\frac{1}{4} z = x_h + y_h \qquad \text{(crude oil} \rightarrow \text{Ho)}$$

$$t_a y_a + t_h y_h \le T \qquad \text{(cracker time)}$$

$$z \le B \qquad \text{(available crude oil)}$$

$$x_a, y_a, x_h, y_h, z \geq 0.$$

P5. The objective fn. vector $\vec{c} = (3, 11, -8, 0)$

a. $\vec{d}^{w_4} = (1, 0, -4, 1)$ does NOT lead to a conclusion that the LP is unbounded, since its components are not all nonnegative.

b. $\vec{d}^{w_4} = (1, 3, 0, 1)$ has associated reduced cost $\bar{c}_{w_4} = 36$. Since the LP is minimizing, $\vec{d}^{w_4}$ is not improving. So, even though all components of $\vec{d}^{w_4}$ are nonnegative, we cannot conclude that the LP is unbounded.

c. $\vec{d}^{w_4} = (1, 0, 3, 1)$ has associated reduced cost $\bar{c}_{w_4} = -21$, and so $\vec{d}^{w_4}$ is improving, with all nonnegative components. Therefore, we can conclude that the LP is unbounded.

d. $\vec{d}^{w_4} = (-1, 1, -2, 1)$. Similar to part a.

P6. Symbolic input parameters:

P = set of presents
C = set of children
v_{ij} = happiness of child i w/present j for $i \in C$, $j \in P$
b_j = # present j available for $j \in P$.

Decision variables: x_{ij} = # present j given to child i for $i \in C$, $j \in P$.

Model:

$$\max \quad \min \Big\{ \underbrace{\sum_{j \in P} v_{ij} x_{ij}}_{\text{happiness of child } i} : i \in C \Big\}$$

$$\text{s.t.} \quad \sum_{i \in C} x_{ij} \le b_j \quad \text{for } j \in P \quad \text{(available presents)}$$

$$x_{ij} \ge 0 \quad \text{for } i \in C, j \in P.$$

convert to LP

$$\max \quad z$$

$$\text{s.t.} \quad z \le \sum_{j \in P} v_{ij} x_{ij} \quad \text{for } i \in C$$

$$\sum_{i \in C} x_{ij} \le b_j \quad \text{for } j \in P$$

$$x_{ij} \ge 0 \quad \text{for } i \in C, j \in P.$$