Name:

SA305 – Linear Programming   Spring 2014
Asst. Prof. Nelson Uhan

Exam 1

Instructions

- You have until the end of the class period to complete this exam.

- There are 3 problems on this exam, worth a total of 100 points.

- You may <u>not</u> consult any outside materials (e.g. notes, textbooks, homework).

- You may <u>not</u> use a calculator.

- **Show all your work.** Your answers should be legible and clearly labeled. It is <u>your</u> responsibility to make sure that I understand what you are doing. You will be awarded partial credit if your work merits it.

- Keep this booklet intact.

Additional page for answers or scratchwork

Problem 1. (40 total points)

The Simplexville Emergency Operations Center has asked you to help schedule its phone operators. Based on historical data, the center has determined that it needs the following numbers of operators throughout the day:

Time period	0000-0400	0400-0800	0800-1200	1200-1600	1600-2000	2000-0000
Minimum # operators needed	10	8	20	18	25	13

Operators work 12-hour shifts starting at 0000, 0400, 0800, 1200, 1600, and 2000. One operator working a shift that starts at 0400, 0800, or 1200 costs a total of $300, while one operator working a shift that starts at 1600, 2000, or 0000 costs a total of $400.

a. (35 points) Write a linear program that determines how to schedule operators at minimum cost. Assume fractional solutions are acceptable. Be sure to briefly

- define any symbolic input parameters you use,
- define the decision variables that you use, and
- explain the objective function, general constraints, and variable bounds that you write.

For this problem, you may use symbolic input parameters, but you do not have to.

b. (5 points) Due to union regulations, at most 30% of the operators can work a shift that starts at 1600, 2000, or 0000. How would you change your linear program in part a to enforce this?

Problem 2. (40 total points)

The operations research analyst at the Mighty Metal Workshop has asked you to help with a problem. The workshop needs to cut at least 37 large disks and 21 small disks from sheet metal rectangles of a standard size. Three cutting patterns are available, which yield the following numbers of disks and waste:

	Large disks	Small disks	Waste (m^2)
Pattern 1	3	0	0.34
Pattern 2	0	5	0.22
Pattern 3	1	3	0.27

The workshop would like to minimize the waste produced from cutting disks. The analyst has started writing a linear program using the following symbolic input parameters:

$$D = \text{set of disk types} = \{\text{Large, Small}\}$$
$$P = \text{set of patterns} = \{1, 2, 3\}$$
$$r_i = \text{minimum number of type } i \text{ disks needed} \qquad \text{for } i \in D$$
$$w_j = \text{m}^2 \text{ of waste produced by pattern } j \qquad \text{for } j \in P$$
$$a_{ij} = \text{number of type } i \text{ disks produced by pattern } j \quad \text{for } i \in D \text{ and } j \in P$$

The analyst has also defined the decision variables

$$x_j = \text{number of sheet metal rectangles cut with pattern } j \quad \text{for } j \in P$$

a. (10 points) Write the objective using the analyst's notation. Use summations and for statements if appropriate.

b. (20 points) Write the constraints using the analyst's notation. Use summations and for statements if appropriate.

c. (10 points) Suppose that pattern 3 wears down the workshop's machines quickly, so the machinists insist that for every rectangle cut with pattern 3, there must be at least 3 rectangles cut with either pattern 1 or pattern 2. How would you change the linear program you wrote above to account for this?

Problem 3. (20 total points = 5 points × 4 parts)

Consider the following feasible region of a linear program and its graph:

$$3x - y \le 6$$
$$x + 3y \le 12$$
$$x \ge 0$$
$$y \ge 0$$
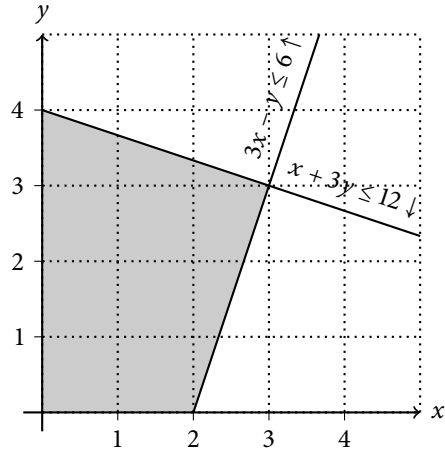
a. Suppose the objective is to minimize $x + y$. Give (i) an optimal solution and (ii) the optimal value.

b. Give a constraint that you could add to this linear program to make it infeasible.

Consider the following sets:

$$S = \{1, 2, 3, 4\} \quad T_1 = \{a, e\} \quad T_2 = \{b, c, d\} \quad T_3 = \{a, c\} \quad T_4 = \{b, e\}$$

c. <u>Without</u> summations and for statements, write an equivalent expression to $\displaystyle\sum_{i \in S: c \in T_i} y_i$.

d. Using summations and for statements, write the following <u>as compactly as possible</u>:

$$f_{a1} y_1 + f_{a2} y_2 + f_{a3} y_3 + f_{a4} y_4 = g_a$$
$$f_{e1} y_1 + f_{e2} y_2 + f_{e3} y_3 + f_{e4} y_4 = g_e$$

5

Additional page for answers or scratchwork