Lesson 12. Multiperiod Models, Revisited

Example 1. Let $T = \{1, 2, 3\}$ and $V = \{S, W\}$. Write the following expressions as compactly as possible using summations and for statements.

a. $$x_{S1} + x_{W1} \leq b$$
$$x_{S2} + x_{W2} \leq b$$
$$x_{S3} + x_{W3} \leq b$$

b. $$y_{S0} + x_{S1} = d_{S1} + y_{S1}$$
$$y_{S1} + x_{S2} = d_{S2} + y_{S2}$$
$$y_{S2} + x_{S3} = d_{S3} + y_{S3}$$

Example 2. Priceler manufactures sedans and wagons. The demand for each type of vehicle in the next three months is:

	Month 1	Month 2	Month 3
Sedans	1100	1500	1200
Wagons	600	700	500

Assume that the demand for both vehicles must be met exactly each month. Each sedan costs \$2000 to produce, and each wagon costs \$1500 to produce. Vehicles not sold in a given month can be held in inventory. To hold a vehicle in inventory from one month to the next costs \$150 per sedan and \$200 per wagon. During each month, at most 1500 vehicles can be produced. At the beginning of month 1, there are 200 sedans and 100 wagons available. Write a linear program that can be used to minimize Priceler's costs during the next three months.

Recall that back in Lesson 9, we wrote the following linear program for this problem without symbolic input parameters and with slightly different variable names:

Decision variables.

$$x_{S1} = \text{number of sedans to produce in month 1}$$
$$x_{W1} = \text{number of wagons to produce in month 1}$$
$$y_{S1} = \text{number of sedans to hold in inventory at the end of month 1}$$
$$y_{W1} = \text{number of wagons to hold in inventory at the end of month 1}$$
$$x_{S2}, x_{S3}, x_{W2}, x_{W3}, y_{S2}, y_{S3}, y_{W2}, y_{W3} \text{ defined similarly}$$

Objective function and constraints.

$$
\begin{aligned}
\text{minimize} \quad & 2000(x_{S1} + x_{S2} + x_{S3}) + 1500(x_{W1} + x_{W2} + x_{W3}) \\
& \quad + 150(y_{S1} + y_{S2} + y_{S3}) + 200(y_{W1} + y_{W2} + y_{W3}) \quad \text{(total cost)} \\
\text{subject to} \quad & \left.\begin{aligned} x_{S1} + x_{W1} &\le 1500 \\ x_{S2} + x_{W2} &\le 1500 \\ x_{S3} + x_{W3} &\le 1500 \end{aligned}\right\} \text{(monthly production capacity)} \\
& \left.\begin{aligned} 200 + x_{S1} &= 1100 + y_{S1} \\ y_{S1} + x_{S2} &= 1500 + y_{S2} \\ y_{S2} + x_{S3} &= 1200 + y_{S3} \end{aligned}\right\} \text{(sedan balance)} \qquad \left.\begin{aligned} 100 + x_{W1} &= 600 + y_{W1} \\ y_{W1} + x_{W2} &= 700 + y_{W2} \\ y_{W2} + x_{W3} &= 500 + y_{W3} \end{aligned}\right\} \text{(wagon balance)} \\
& \left.\begin{aligned} & x_{S1} \ge 0, x_{S2} \ge 0, x_{S3} \ge 0, y_{S1} \ge 0, y_{S2} \ge 0, y_{S3} \ge 0 \\ & x_{W1} \ge 0, x_{W2} \ge 0, x_{W3} \ge 0, y_{W1} \ge 0, y_{W2} \ge 0, y_{W3} \ge 0 \end{aligned}\right\} \text{(nonnegativity)}
\end{aligned}
$$

Now let's write a linear program using symbolic input parameters: