SA305 – Linear Programming
Asst. Prof. Nelson Uhan

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Lesson 13. Work Scheduling Models, Revisited

1 The postal workers problem, revisited

Example 1. Postal employees in Simplexville work for 5 consecutive days, followed by 2 days off, repeated weekly. Below are the minimum number of employees needed for each day of the week:

Day	Employees needed
Monday	7
Tuesday	8
Wednesday	7
Thursday	6
Friday	6
Saturday	4
Sunday	5

We want to determine the minimum total number of employees needed.

Our original model:

Decision variables. Let

$$\begin{aligned} x_1 &= \text{number of employees who work “shift 1” – i.e. Monday to Friday} \\ x_2 &= \text{number of employees who work “shift 2” – i.e. Tuesday to Saturday} \\ &\vdots \\ x_7 &= \text{number of employees who work “shift 7” – i.e. Sunday to Thursday} \end{aligned}$$

Objective function and constraints.

$$\begin{array}{rlr} \min & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 & \\ \text{s.t.} & x_1 + x_4 + x_5 + x_6 + x_7 \geq 7 & \text{(Mon)} \\ & x_1 + x_2 + x_5 + x_6 + x_7 \geq 8 & \text{(Tue)} \\ & x_1 + x_2 + x_3 + x_6 + x_7 \geq 7 & \text{(Wed)} \\ & x_1 + x_2 + x_3 + x_4 + x_7 \geq 6 & \text{(Thu)} \\ & x_1 + x_2 + x_3 + x_4 + x_5 \geq 6 & \text{(Fri)} \\ & x_2 + x_3 + x_4 + x_5 + x_6 \geq 4 & \text{(Sat)} \\ & x_3 + x_4 + x_5 + x_6 + x_7 \geq 5 & \text{(Sun)} \\ & x_1, \; x_2, \; x_3, \; x_4, \; x_5, \; x_6, \; x_7 \geq 0 & \end{array}$$

- Left hand side of (Mon): add up the variables x_i <u>such that shift i covers Monday</u>
- We need a way to specify elements of a set that meet certain characteristics

2 Some more set notation

- What if we only want certain elements of a set?
- " : " notation

$$j \in S : [\text{condition}] \quad \Leftrightarrow \quad j \in \text{elements of } S \text{ } \underline{\text{such that}} \text{ [condition] holds}$$

- For example:
    - Define $N = \{1, 2, 3\}$, $S_1 = \{a, b\}$, $S_2 = \{b, c\}$, $S_3 = \{a, c\}$
    - Then
        - $j \in N : j \geq 2 \quad \Leftrightarrow$
        - $j \in N : a \in S_j \quad \Leftrightarrow$
- Some people use " | " instead ":"

Describe the input parameters of Example 1 using sets and for statements.

Write a linear program for Example 1 using the symbolic input parameters you described above.

3 GMPL code

GMPL model file for this linear program:

```
# Model for postal employees problem in Lesson 13

## Input parameters ##
set days;                       # days of the week
set shifts;                     # shifts
set shift_days{j in shifts};    # days worked for each shift
param required{i in days};      # number of employees needed for each day

## Decision variables and variable bounds ##
var x{j in shifts} >= 0;        # number of employees assigned to each shift

## Objective function ##
# Minimize total number of employees
minimize total_employees: sum{j in shifts} x[j];

## General constraints ##
# Number of employees working on day i >= minimum required on day i
subject to employees_needed{i in days}:
  sum{j in shifts: i in shift_days[j]} x[j] >= required[i];

end;
```

GMPL data file for this linear program:

```
# Days of the week
set days := Mon Tue Wed Thu Fri Sat Sun;

# Shifts
set shifts := 1 2 3 4 5 6 7;

# Days worked in each shift
set shift_days[1] := Mon Tue Wed Thu Fri;
set shift_days[2] := Tue Wed Thu Fri Sat;
set shift_days[3] := Wed Thu Fri Sat Sun;
set shift_days[4] := Thu Fri Sat Sun Mon;
set shift_days[5] := Fri Sat Sun Mon Tue;
set shift_days[6] := Sat Sun Mon Tue Wed;
set shift_days[7] := Sun Mon Tue Wed Thu;

# Number of employees needed for each day
param required :=
  Mon   7
  Tue   8
  Wed   7
  Thu   6
  Fri   6
  Sat   4
  Sun   5;

end;
```

4 If we have time...

Example 2. Suppose instead that in a given week, postal employees in Simplexville work for 2 consecutive days followed by 1 day off, and then work for 3 consecutive days followed by 1 day off. For example, an employee might work Monday and Tuesday, have Wednesday off, then work Thursday, Friday and Saturday, and have Sunday off. How would you change the GMPL code on the previous page to accommodate this?