Lesson 29. Introduction to Integer Linear Programs: Timetabling Problems

- A decision variable is **integral** if it is restricted to a specified interval of integers

  - Special example: **binary** variables are integral

$$x_j \in \{0,1\} \quad = \quad \begin{cases} x_j \leq 1 \\ x_j \geq 0 \\ x_j \text{ integer} \end{cases}$$

  - Binary variables are especially useful for modeling, for example:
    - Yes/No decisions
    - Logical constraints

- An optimization model is an **integer linear program** if

  - the objective function is linear

  - the constraints are linear

  - at least one of its decision variables is integral

- The math department at USNA uses integer programming to assign instructors to courses, meeting times and rooms

  - This coming Fall semester, there are 33 courses, 184 sections, 69 instructors, and 30 rooms

  - Results in an integer linear program with 140,617 variables and 207,272 constraints

- In this lesson, we'll look at a similar, but simpler example

- Note: in GMPL, restricting variables to be binary or integer is easy:

```
var x{j in J} binary;
var y >= 0, integer;
```

Example 1. The math department at Simplexville University has four professors who teach operations research courses. In the coming Fall semester, the department is offering Integer Programming (IP), Stochastic Processes (SP), and Simulation Modeling (SM). These courses can be taught during first or second period, when there are two rooms available, or third period, when there is just one room available. Each professor's teaching assignment and time preferences (higher is better) are given below:

	Professor Ann	Professor Bob	Professor Carol	Professor David
Course	IP	SM	SP	IP
Period 1	1	1	2	5
Period 2	3	1	5	3
Period 3	5	5	4	1

Since Professor Ann and Professor David are both teaching Integer Programming, their sections should not be scheduled for the same period, so that students have more flexibility in creating their schedules.

Formulate the problem of assigning professors to periods in a way that maximizes the total preference points of the professors as an integer program.

Input parameters.

Decision variables.

(maximize total preference points)

(each instructor/class → exactly one period)

(room availability in each period)

(Ann and David cannot teach simultaneously)

(binary variables)

Food for thought: how would you model the following constraints?

- If Professor Ann teaches first period, then Professor Bob must teach third period.
- If Professor Carol does not teach second period, then Professor David cannot teach second period.