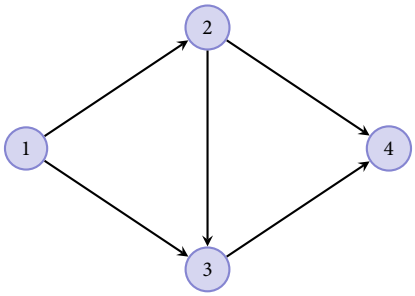
Lesson 30. Introduction to Networks and the Shortest Path Problem

1 Graphs and networks

- This lesson: what is the shortest way to get from Point A to Point B?
- **Graphs** model how various entities are connected
- A **directed graph** or **network** (N, A) consists of
  - set of **nodes** N (also known as **vertices**)
  - set of **arcs** A
    - arcs are directed from one node to another
    - arc from node i to node j is denoted by (i, j)

Example 1.



- Networks are everywhere
  - Physical networks: road networks, airline traffic networks
  - Abstract networks: organizational charts, precedence relationships in projects
  - Others?

2 Paths and the shortest path problem

- A **path** is a sequence of arcs connecting two specified nodes in a graph:
  - Each arc must have exactly one node in common with its predecessor in the sequence
  - Arcs must be passed in the forward direction
  - No node may be visited more than once

Example 2. Give an example of a path from node 1 to node 4 in the network in Example 1.

- **The shortest path problem**
  - Suppose we are given a network (N, A), and each arc (i, j) in A has a **length** (or cost) c_{ij}
  - Designate one node in the network as the **origin** s, and another as the **destination** t
    - Assume that the origin only has outgoing arcs, and the destination only has incoming arcs
  - What is the shortest path from s to t?
- Input parameters:

$$N = \text{set of nodes}$$
$$A = \text{set of arcs}$$
$$c_{ij} = \text{length of arc } (i, j) \quad \text{for } (i, j) \in A$$

- Decision variables:

- Objective – minimize the length of the selected path:

- Constraints – exactly 1 arc out of the origin must be selected:

- Exactly 1 arc into the destination must be selected:

- For all nodes other than the origin and destination, "what goes in = what goes out":

- Binary variables: