Lesson 4. Blending Models

Example 1. The Hoosier Gasoline Company produces two blends of gasoline, regular and premium, by mixing three different types of oil. Each type of oil comes in barrels and has its own costs and octane ratings, which are given below:

Type	Cost/Barrel	Octane Rating
1	45	93
2	35	90
3	20	87

Premium gasoline must consist of at least 30% Type 1 oil. In addition, the minimum weighted average octane rating and minimum production requirements for each blend are as follows:

Blend	Weighted Average Octane Rating	Demand
Regular	89	15,000 barrels
Premium	91	12,500 barrels

Formulate a linear program that determines how to meet the demand for each blend of gasoline at minimum cost.

Ex. 2 barrels of Type 1, 1 barrel of Type 2

$\Rightarrow$ 3 barrels of blended gas

Weighted average octane rating $= \frac{2}{3}(93) + \frac{1}{3}(90)$

DVs: R_1 = # barrels of Type 1 used in regular

P_1 = # barrels of Type 1 used in premium

R_2, P_2, R_3, P_3 defined similarly

fraction of regular that is Type 1

$$93 \frac{R_1}{R_1+R_2+R_3} + 90 \frac{R_2}{R_1+R_2+R_3} + 87 \frac{R_3}{R_1+R_2+R_3} \geq 89$$

minimize $45(R_1+P_1) + 35(R_2+P_2) + 20(R_3+P_3)$ (total cost)

subject to $P_1 \geq 0.3(P_1+P_2+P_3)$ (premium has at least 30% Type 1)

$93R_1 + 90R_2 + 87R_3 \geq 89(R_1+R_2+R_3)$ (regular octane)

$93P_1 + 90P_2 + 87P_3 \geq 91(P_1+P_2+P_3)$ (premium octane)

$R_1 + R_2 + R_3 \geq 15000$ (regular demand)

$P_1 + P_2 + P_3 \geq 12500$ (premium demand)

$R_1 \geq 0,\ R_2 \geq 0,\ R_3 \geq 0$

$P_1 \geq 0,\ P_2 \geq 0,\ P_3 \geq 0$ (nonnegativity)

Example 2. You are a portfolio manager in charge of a bank portfolio with \$10 million to invest. There are 4 different securities available:

Bond name	Bond type	Years to maturity	Rate of return at maturity (%)
1	Municipal	9	4.3
2	Agency	15	2.7
3	Government	4	2.5
4	Government	3	2.2

The bank has some policies that limit how you can construct your portfolio:

1. Municipal and agency bonds must total at least \$4 million
2. The (weighted) average years to maturity of the portfolio must not exceed 6 years
3. Bonds cannot be "shorted" (cannot buy negative amounts of bonds)

$$\frac{9x_1}{x_1+x_2+x_3+x_4} + \frac{15x_2}{x_1+x_2+x_3+x_4} + \frac{4x_3}{x_1+x_2+x_3+x_4} + \frac{3x_4}{x_1+x_2+x_3+x_4} \le 6$$

Write a linear program that determines a portfolio of the above securities that maximizes earnings.

DVs: x_1 = amount to invest in Bond 1, in millions

x_2, x_3, x_4 defined similarly.

maximize $0.043x_1 + 0.027x_2 + 0.025x_3 + 0.022x_4$ (total earnings)

subject to $x_1 + x_2 \ge 4$ (municipal + agency req.)

$9x_1 + 15x_2 + 4x_3 + 3x_4 \le 6(x_1+x_2+x_3+x_4)$ (YTM req.)

also ok: $x_1, x_2, x_3, x_4 \ge 0$ $\quad$ $x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0,\ x_4 \ge 0$ (nonnegativity / bonds can't be shorted)

$x_1 + x_2 + x_3 + x_4 \le 10$ (budget)