Lesson 5. Work Scheduling Models

Example 1. Postal employees in Simplexville work for 5 consecutive days, followed by 2 days off, repeated weekly. Below are the minimum number of employees needed for each day of the week:

Day	Employees needed
Monday (1)	7
Tuesday (2)	8
Wednesday (3)	7
Thursday (4)	6
Friday (5)	6
Saturday (6)	4
Sunday (7)	5

Write a linear program that determines the minimum total number of employees needed. You may assume that fractional solutions are acceptable.

Potential DVs: y_1 = # employees who work on day 1
$y_2, \dots, y_7$ defined similarly

how do we ensure employees work for 5 consecutive days, then 2 days off?

DVs: x_1 = # employees who work on days 1–5 (Mon–Fri)
x_2 = # employees who work on days 2–6 (Tue–Sat)
x_3, x_4, x_5, x_6, x_7 defined similarly

$$
\begin{aligned}
\text{minimize} \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 && \text{(total \# employees)}\\
\text{subject to} \quad & x_1 + x_4 + x_5 + x_6 + x_7 \geq 7 && \text{(Mon)}\\
& x_1 + x_2 + x_5 + x_6 + x_7 \geq 8 && \text{(Tue)}\\
& x_1 + x_2 + x_3 + x_6 + x_7 \geq 7 && \text{(Wed)}\\
& x_1 + x_2 + x_3 + x_4 + x_7 \geq 6 && \text{(Thu)}\\
& x_1 + x_2 + x_3 + x_4 + x_5 \geq 6 && \text{(Fri)}\\
& x_2 + x_3 + x_4 + x_5 + x_6 \geq 4 && \text{(Sat)}\\
& x_3 + x_4 + x_5 + x_6 + x_7 \geq 5 && \text{(Sun)}\\
& x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0 && \text{(nonnegativity)}\\
& x_5 \geq 0,\ x_6 \geq 0,\ x_7 \geq 0
\end{aligned}
$$

Example 2. At the Rusty Knot, tables are set and cleared by runners working 5-hour shifts that start on the hour, from 5am to 10am. Runners in these 5-hour shifts take a mandatory break during the 3rd hour of their shifts. For example, the shift that starts at 9am ends at 2pm, with a break from 11am-12pm. The Rusty Knot pays $7 per hour for the shifts that start at 5am, 6am, and 7am, and $6 per hour for the shifts that start at 8am, 9am, and 10am. Past experience indicates that the following number of runners are needed at each hour of operation:

	Hour	Number of runners required
1	5am-6am	2
2	6am-7am	3
3	7am-8am	5
4	8am-9am	5
5	9am-10am	4
6	10am-11am	3
	11am-12pm	6
	12pm-1pm	4
	1pm-2pm	3
	2pm-3pm	2

Formulate a linear program that determines a cost-minimizing staffing plan. You may assume that fractional solutions are acceptable.

DVs:
x_1 = # runners who work the shift starting at 5am (5am – 10am)
x_2 = # runners who work the shift starting at 6am (6am – 11am)
x_3, x_4, x_5, x_6 defined similarly

minimize $7(5)(x_1 + x_2 + x_3) + 6(5)(x_4 + x_5 + x_6)$ (total cost)

subject to
$x_1 \geq 2$ (5am – 6am)
$x_1 + x_2 \geq 3$ (6am – 7am)
$x_2 + x_3 \geq 5$ (7am – 8am)
$x_1 + x_3 + x_4 \geq 5$ (8am – 9am)
$x_1 + x_2 + x_4 + x_5 \geq 4$ (9am – 10am)
$x_2 + x_3 + x_5 + x_6 \geq 3$ (10am – 11am)
$x_3 + x_4 + x_6 \geq 6$ (11am – 12pm)
$x_4 + x_5 \geq 4$ (12pm – 1pm)
$x_5 + x_6 \geq 3$ (1pm – 2pm)
$x_6 \geq 2$ (2pm – 3pm)

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,$
$x_4 \geq 0,\ x_5 \geq 0,\ x_6 \geq 0$ (nonnegativity)