Lesson 6. Sets, Summations, For Statements

1 Sets

- A **set** is a collections of elements/objects, e.g.

$$S = \{1, 2, 3, 4, 5\} \qquad \text{Fruits} = \{\text{Apple}, \text{Orange}, \text{Pear}\} \tag{1}$$

- "in" symbol:

$$i \in N \quad \Leftrightarrow \quad \text{"element } i \text{ is in the set } N\text{"}$$

- For example:

2 Summations

- Summation symbol over sets:

$$\sum_{i \in N} \quad \Leftrightarrow \quad \text{"sum over all elements of } N\text{"}$$

- For example:

- Common shorthand: if $N = \{1, 2, \ldots, n\}$, then

$$\sum_{i \in N} \text{ is the same as } \sum_{i \in \{1,2,\ldots,n\}} \text{ as well as } \sum_{i=1}^{n}$$

Example 1. Let the sets S and Fruits be defined as above in (1). Write each of the following as compactly as possible using summation notation:

a. $x_{\text{Apple}} + x_{\text{Orange}} + x_{\text{Pear}}$
b. $1y_1 + 2y_2 + 3y_3 + 4y_4 + 5y_5$

3 For statements

- "for" statements over sets:

$$\text{for } i \in N \quad \Leftrightarrow \quad \text{"repeat for each element of } N\text{"}$$

- For example:

$$c_j x_1 + d_j x_2 \le b_j \quad \text{for } j \in \text{Fruits} \quad \Leftrightarrow$$

- Common shorthand: if $N = \{1, 2, \dots, n\}$, then

"for $i \in N$" is the same as "for $i \in \{1, 2, \dots, n\}$" as well as "for $i = 1, 2, \dots, n$"

- Sometimes we say "for all $i \in N$" instead of "for $i \in N$"

4 Multiple indices

- Sometimes it may be useful to use decision variables with multiple indices
- Example:
  - Set of hat types: $H = \{A, B, C\}$
  - Set of factories: $F = \{1, 2\}$
  - Each hat type can be be produced at each factory
  - Define decision variables:

$$x_{i,j} = \text{number of type } i \text{ hats produced at factory } j \qquad \text{for } i \in H \text{ and } j \in F \tag{2}$$

  - What decision variables have we just defined? How many are there?

Example 2. Using the decision variables defined in (2), write expressions for

a. Total number of type C hats produced
b. Total number of hats produced at facility 2

Use summation notation if possible.

- Suppose

$$c_{i,j} = \text{cost of producing one type } i \text{ hat at factory } j \qquad \text{for } i \in H \text{ and } j \in F$$

- If we produce $x_{i,j}$ hats of type i at factory j (for $i \in H$ and $j \in F$), then the total cost is

Example 3. Let $M = \{1, 2, 3\}$ and $N = \{1, 2, 3, 4\}$. Write the following as compactly as possible using summation notation and "for" statements.

$$\begin{aligned} \text{Let } y_1 &= \text{amount of product 1 produced} \\ y_2 &= \text{amount of product 2 produced} \\ y_3 &= \text{amount of product 3 produced} \\ y_4 &= \text{amount of product 4 produced} \end{aligned}$$

$$\begin{aligned} a_{1,1}y_1 + a_{1,2}y_2 + a_{1,3}y_3 + a_{1,4}y_4 &= b_1 \\ a_{2,1}y_1 + a_{2,2}y_2 + a_{2,3}y_3 + a_{2,4}y_4 &= b_2 \\ a_{3,1}y_1 + a_{3,2}y_2 + a_{3,3}y_3 + a_{3,4}y_4 &= b_3 \end{aligned}$$