Lesson 8. Work Scheduling Models, Revisited

1 The postal workers problem, revisited

Example 1. Postal employees in Simplexville work for 5 consecutive days, followed by 2 days off, repeated weekly. Below are the minimum number of employees needed for each day of the week:

Day	Employees needed
Monday	7
Tuesday	8
Wednesday	7
Thursday	6
Friday	6
Saturday	4
Sunday	5

We want to determine the minimum total number of employees needed.

Our original model:

Decision variables. Let

$$\begin{aligned} x_1 &= \text{number of employees who work ``shift 1'' – i.e. Monday to Friday} \\ x_2 &= \text{number of employees who work ``shift 2'' – i.e. Tuesday to Saturday} \\ &\vdots \\ x_7 &= \text{number of employees who work ``shift 7'' – i.e. Sunday to Thursday} \end{aligned}$$

Objective function and constraints.

$$\begin{aligned} \min \quad & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 & & \\ \text{s.t.} \quad & x_1 + x_4 + x_5 + x_6 + x_7 \geq 7 & & \text{(Mon)} \\ & x_1 + x_2 + x_5 + x_6 + x_7 \geq 8 & & \text{(Tue)} \\ & x_1 + x_2 + x_3 + x_6 + x_7 \geq 7 & & \text{(Wed)} \\ & x_1 + x_2 + x_3 + x_4 + x_7 \geq 6 & & \text{(Thu)} \\ & x_1 + x_2 + x_3 + x_4 + x_5 \geq 6 & & \text{(Fri)} \\ & x_2 + x_3 + x_4 + x_5 + x_6 \geq 4 & & \text{(Sat)} \\ & x_3 + x_4 + x_5 + x_6 + x_7 \geq 5 & & \text{(Sun)} \\ & x_1, \quad x_2, \quad x_3, \quad x_4, \quad x_5, \quad x_6, \quad x_7 \geq 0 & & \end{aligned}$$

- Left hand side of (Mon): add up the variables x_i such that shift i covers Monday
- We need a way to specify elements of a set that meet certain characteristics

2 Some more set notation

- What if we only want certain elements of a set?
- " : " notation

$$j \in S : [\text{condition}] \quad \Leftrightarrow \quad j \in \text{elements of } S \text{ such that } [\text{condition}] \text{ holds}$$

- For example:
  - Define $N = \{1, 2, 3\}$, $S_1 = \{a, b\}$, $S_2 = \{b, c\}$, $S_3 = \{a, c\}$
  - Then
    - $j \in N : j \geq 2 \quad \Leftrightarrow$
    - $j \in N : a \in S_j \quad \Leftrightarrow$
- Some people use " | " instead ":"
- Describe the constants of Example 1 using sets and parameters.
- Write a parameterized linear program for Example 1 using the sets and parameters you described above.

3 The Rusty Knot, revisited

Example 2. At the Rusty Knot, tables are set and cleared by runners working 5-hour shifts that start on the hour, from 5am to 10am. Runners in these 5-hour shifts take a mandatory break during the 3rd hour of their shifts. For example, the shift that starts at 9am ends at 2pm, with a break from 11am-12pm. The Rusty Knot pays $7 per hour for the shifts that start at 5am, 6am, and 7am, and $6 per hour for the shifts that start at 8am, 9am, and 10am. Past experience indicates that the following number of runners are needed at each hour of operation:

Hour	Number of runners required
5am-6am	2
6am-7am	3
7am-8am	5
8am-9am	5
9am-10am	4
10am-11am	3
11am-12pm	6
12pm-1pm	4
1pm-2pm	3
2pm-3pm	2

Formulate a linear program that determines a cost-minimizing staffing plan. You may assume that fractional solutions are acceptable.