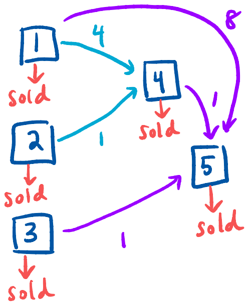
Lesson 10. Production Process Models

Example 1. Midville Manufacturing assembles heavy-duty handling carts. Each cart consists three components: wheels, steering yokes, and carrying platforms. These components are first assembled separately. Then each steering yoke is equipped with 4 wheels to form the front-end subassembly. Finally, front-end subassemblies are combined with 1 carrying platform and 8 additional wheels to complete the cart.

Components, subassemblies, and finished carts require the following amounts of assembly time, and can be sold at the following prices:



Index	Item	Assembly time per unit (hrs)	Price per unit ($)
1	Wheels	0.06	120
2	Steering yokes	0.07	40
3	Carrying platform	0.04	75
4	Front-end subassembly	0.12	400
5	Finished carts	0.32	700

There are 1150 hours of assembly time available.

Write a linear program that determines a production plan for Midville Manufacturing that maximizes its revenue.

Sets. P = set of products = $\{1, 2, 3, 4, 5\}$

Params. a_i = unit assembly time for product i for $i \in P$

p_i = unit price for product i for $i \in P$

DVs. x_i = # product i made for $i \in P$

y_i = # product i sold for $i \in P$

$$\max \quad \sum_{i \in P} p_i y_i \qquad \text{(total revenue)}$$

$$\text{s.t.} \quad \sum_{i \in P} a_i x_i \le 1150 \qquad \text{(assembly time capacity)}$$

$$x_1 = y_1 + 4x_4 + 8x_5 \qquad \text{(wheels)}$$

$$x_2 = y_2 + x_4 \qquad \text{(steering yoke)}$$

$$x_3 = y_3 + x_5 \qquad \text{(carrying platforms)}$$

$$x_4 = y_4 + x_5 \qquad \text{(FES)}$$

$$x_5 = y_5 \qquad \text{(finished carts)}$$

$$x_i \ge 0, \quad y_i \ge 0 \quad \text{for } i \in P \qquad \text{(nonnegativity)}$$

wrong constraints:

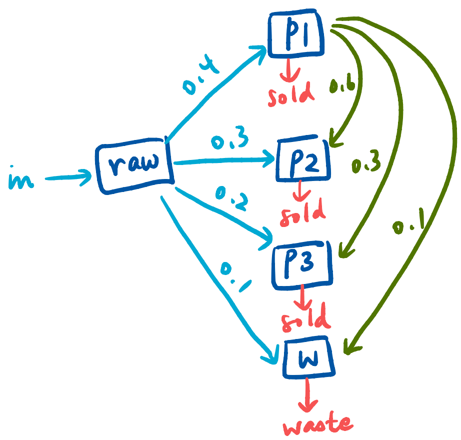
$\overset{1}{x_2} + 4\overset{1}{x_1} = \overset{5}{x_4}$

$x_4 + x_3 + 8x_1 = x_5$

PLUG IN TEST NUMBERS!

Start w/ "smallest" components

Example 2. Alvin Fine produces three perfumes from raw material. Thirty thousand ounces of raw material is available. Each ounce of raw material can be transformed into 0.4 ounces of perfume 1, 0.3 ounces of perfume 2, and 0.2 ounces of perfume 3, while 0.1 ounces is lost as waste material. Each ounce of perfume 1 can be further processed into 0.6 ounces of perfume 2, 0.3 ounces of perfume 3, and 0.1 ounces of waste material. Alvin Fine has been contracted to produce at least 4000 ounces of perfume 1, 8000 ounces of perfume 2, and 10000 ounces of perfume 3. Because of its environmental initiatives it wishes to minimize waste material. Formulate a linear program that determines how much perfume to produce while minimizing waste.



Sets. P = set of perfumes = $\{1, 2, 3\}$

Params. d_i = demand for perfume i for $i \in P$

DVs. r = oz. raw material used

w = oz. waste material produced

x_i = oz. perfume i produced and sold for $i \in P$

y_1 = oz. perfume 1 further processed

$$
\begin{array}{lll}
\min & w & \\
\text{s.t.} & r \leq 30000 & \text{(raw availability)} \\
& 0.4r = x_1 + y_1 & (\text{raw} \rightarrow \text{P1}) \\
& 0.3r + 0.6y_1 = x_2 & (\text{raw} + \text{P1} \rightarrow \text{P2}) \\
& 0.2r + 0.3y_1 = x_3 & (\text{raw} + \text{P1} \rightarrow \text{P3}) \\
& 0.1r + 0.1w = w & (\text{raw} \rightarrow \text{waste}) \\
& x_i \geq d_i \quad \text{for } i \in P & \text{(demand)} \\
& x_i \geq 0 \quad \text{for } i \in P & \\
& r, w, y_1 \geq 0 & \text{(nonnegativity)}
\end{array}
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