

Lesson 16.

# Stochastic dynamic programming, cont.

## 1 The problem

Suppose you have \$5,000 to invest, and at the beginning of each of the next 3 years, you have an opportunity to invest in either of two investments: A or B. Both investments have uncertain profits. For an investment of \$5,000, the profits are as follows:

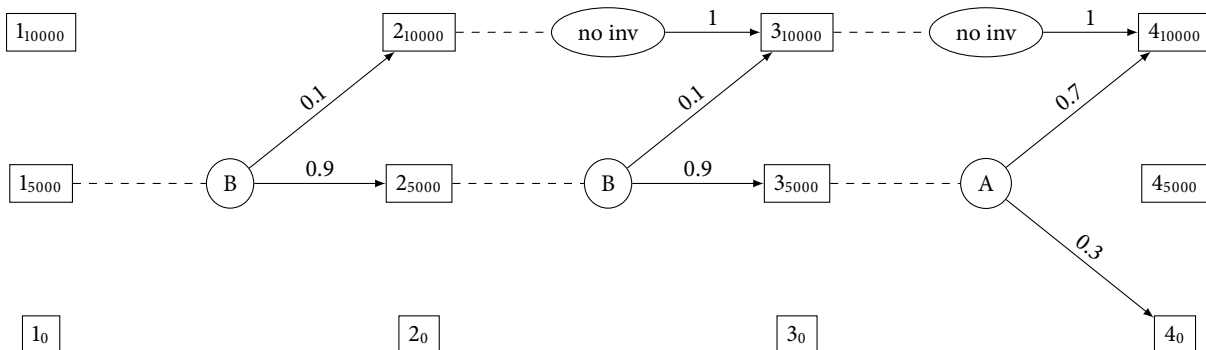
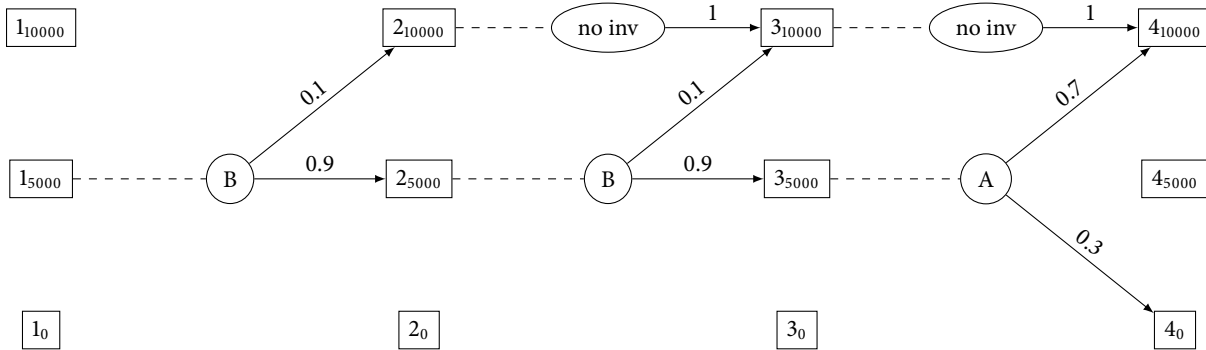
Investment	Profit (\$)	Probability
A	-5,000	0.3
	5,000	0.7
B	0	0.9
	5,000	0.1

You are allowed to make at most one investment each year, and can invest only \$5,000 each time. Any additional money accumulated is left idle.

Formulate a stochastic dynamic program to find an investment policy that maximizes the probability you will have \$10,000 after 3 years.

## 2 Warm up

Consider the following investment policy. What is the probability of having at least \$10,000?



### 3 Formulating the stochastic dynamic program

- Stages:

- States:

- Allowable decisions  $x_t$  at stage  $t$  and state  $n$ :

- Sketch of basic structure – transition probabilities and contributions:

- In words, the value-to-go  $f_t(n)$  at stage  $t$  and state  $n$  is:

- Value-to-go recursion

$$f_t(n) = \min_{x_t \text{ allowable}} \left\{ \sum_{m \text{ state}} p(m | n, t, x_t) [c(m | n, t, x_t) + f_{t+1}(m)] \right\} \text{ for stages } t \text{ and states } n$$

- Boundary conditions:

- Desired value-to-go function value:

- Solving the recursion on  $f_t(n)$ , we obtain:

$t$	$n$	$f_t(n)$	$x_t^*$
1	0	0	no investment
1	5000	0.757	B
1	10000	1	no investment
2	0	0	no investment
2	5000	0.73	B
2	10000	1	no investment
3	0	0	no investment
3	5000	0.7	A
3	10000	1	no investment

- Based on this, what should your investment policy be?

- What is your probability of having \$10000?