

Lesson 14. Stochastic Dynamic Programming, cont.

1 A precision manufacturing problem

Example 1. The Hit-and-Miss Manufacturing Company has received an order to supply one item of a particular type. However, manufacturing this item is difficult, and the customer has specified such stringent quality requirements that the company may have to produce more than one item to obtain an item that is acceptable.

The company estimates that each item of this type will be acceptable with probability $1/2$ and defective with probability $1/2$. Each item costs \$100 to produce, and excess items are worthless. In addition, a setup cost of \$300 must be incurred whenever the production process is setup for this item. The company has time to make no more than 3 production runs, and at most 5 items can be produced in each run. If an acceptable item has not been obtained by the end of the third production run, the manufacturer is in breach of contract and must pay a penalty of \$1600.

The objective is to determine how many items to produce in each production run in order to minimize the total expected cost.

1.1 Warm up

- Suppose the manufacturer produces x items in a single production run.
- What is the probability that at least one of these items is acceptable?

- What is the expected number of acceptable items?

1.2 Modeling the problem

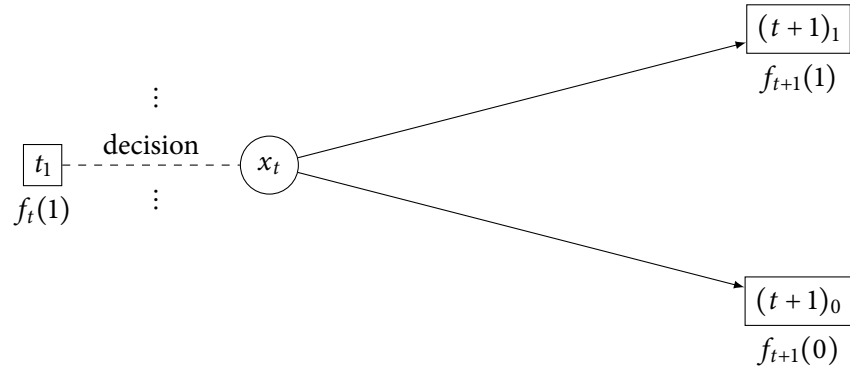
- Stages:

- States:

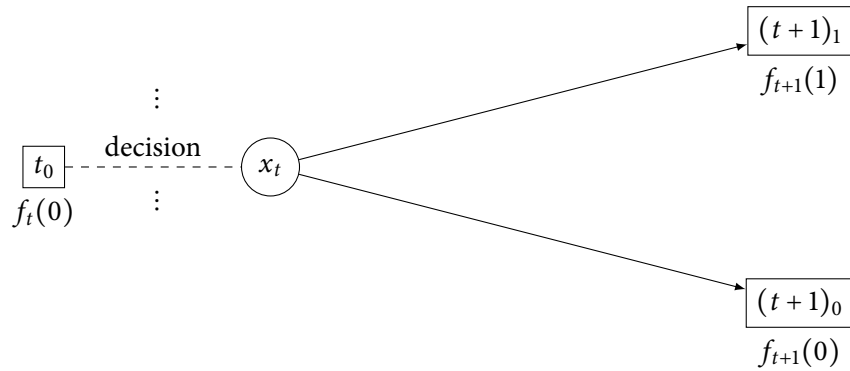
- Allowable decisions x_t at stage t and state n :

- Sketch of basic structure:

- When the state $n = 1$:



- When the state $n = 0$:



- In words, the value-to-go $f_t(n)$ at stage t and state n is:

- Boundary conditions:

- Value-to-go recursion:

$$f_t(n) = \min_{x_t \text{ allowable}} / \max \left\{ \sum_{m \text{ state}} p(m | n, t, x_t) [c(m | n, t, x_t) + f_{t+1}(m)] \right\} \text{ for stages } t \text{ and states } n$$

- Desired value-to-go function value:

1.3 Interpreting the value-to-go function

- Solving the recursion, we get the following value-to-go function values $f_t(n)$ for $t = 1, 2, 3$ and $n = 0, 1$, as well as the decision x_t^* that attained each value:

t	n	$f_t(n)$	x_t^*
1	0	0	0
1	1	675	2
2	0	0	0
2	1	700	2
3	0	0	0
3	1	800	3

- Based on this, what should the company's policy be?

- What is the company's total expected cost?

2 A small investment problem

Example 2. Suppose you have \$5,000 to invest. Over the next 3 years, you want to double your money. At the beginning of each of the next 3 years, you have an opportunity to invest in one of two investments: A or B. Both investments have uncertain profits. For an investment of \$5,000, the profits are as follows:

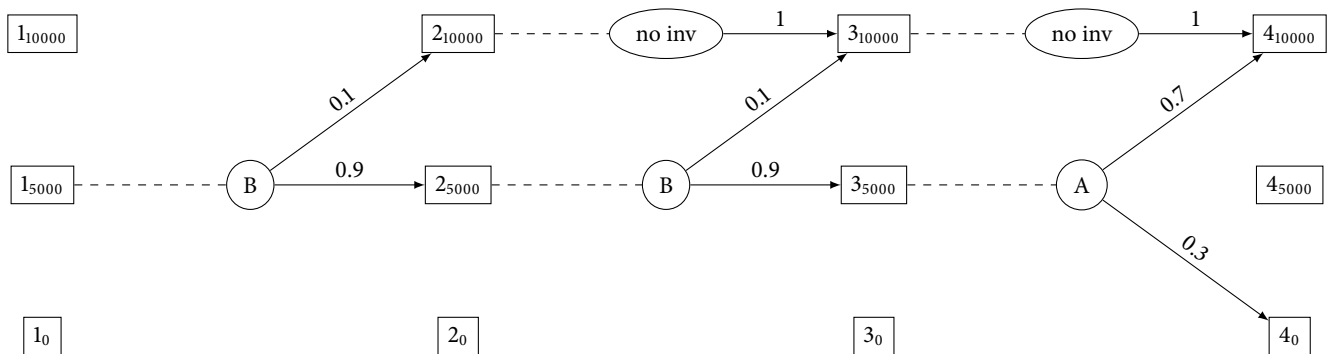
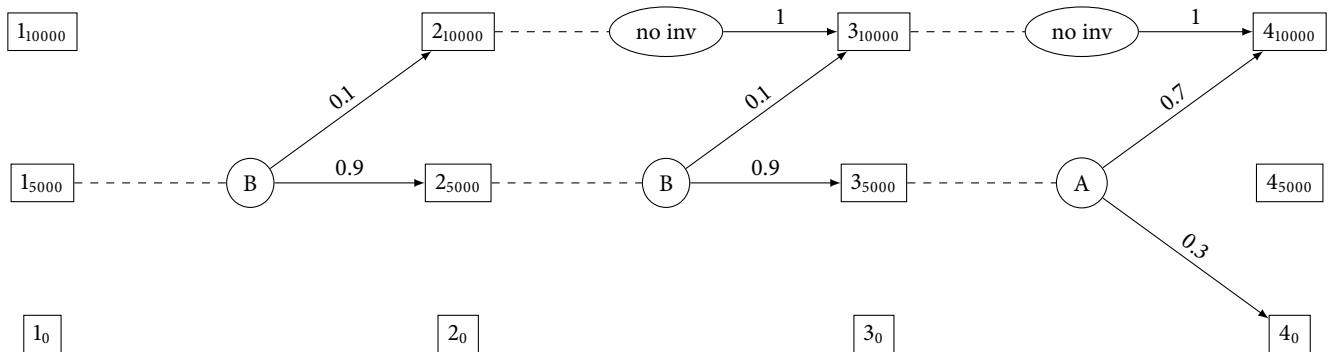
Investment	Profit (\$)	Probability
A	-5,000	0.3
	5,000	0.7
B	0	0.9
	5,000	0.1

You are allowed to make at most one investment each year, and can invest only \$5,000 each time. Any additional money accumulated is left idle. Once you've accumulated \$10,000, you stop investing.

Formulate a stochastic dynamic program to find an investment policy that maximizes the probability you will have \$10,000 after 3 years.

2.1 Warm up

Consider the following investment policy. What is the probability of having at least \$10,000?



2.2 Formulating the stochastic dynamic program

- Stages:

- States:

- Allowable decisions x_t at stage t and state n :

- Sketch of basic structure – transition probabilities and contributions:

- In words, the value-to-go $f_t(n)$ at stage t and state n is:

- Value-to-go recursion

$$f_t(n) = \min / \max_{x_t \text{ allowable}} \left\{ \sum_{m \text{ state}} p(m | n, t, x_t) [c(m | n, t, x_t) + f_{t+1}(m)] \right\} \text{ for stages } t \text{ and states } n$$

- Boundary conditions:

- Desired value-to-go function value:

2.3 Interpreting the value-to-go function

- Solving the recursion on $f_t(n)$, we obtain:

t	n	$f_t(n)$	x_t^*
1	0	0	no investment
1	5000	0.757	B
1	10000	1	no investment
2	0	0	no investment
2	5000	0.73	B
2	10000	1	no investment
3	0	0	no investment
3	5000	0.7	A
3	10000	1	no investment

- Based on this, what should your investment policy be?

- What is your probability of having \$10,000?