Lesson 8. DPs with Multidimensional States

Example 1. You are in charge of determining which subset of the following requirements should be shipped on the next C-17 to another base:

Requirement	Capability Value	Weight (tons)	Volume (m^3)
Large munitions	50	43	250
Small munitions	30	17	130
Food	80	26	370
Medical supplies	40	4	180
Repair parts	70	35	400

The C-17 has a weight capacity of 80 tons, and a volume capacity of 700 m^3. The goal is to maximize the total capability value of the requirements shipped.

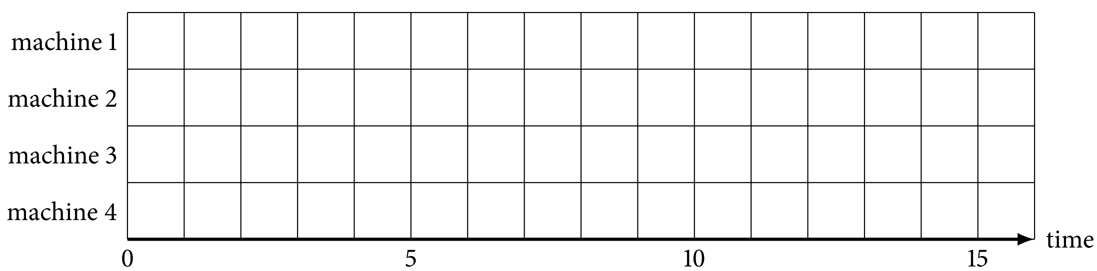
Formulate this problem as a dynamic program by giving its shortest/longest path representation.

Example 2. The Markov Micromanufacturing Company has 9 production jobs it needs to process in the next 24 hours. The company has 4 identical machines that run in parallel. Each of these 9 jobs must be run on one of these machines **nonpreemptively**: that is, once a job is started on a machine, it must stay on that machine until it is completed. The processing times of these jobs are given below:

job	1	2	3	4	5	6	7	8	9
processing time (hours)	7	7	6	6	5	5	4	4	4

The company wants to minimize the **makespan**, or the completion time of the last job to finish processing.

- Let m be the number of machines — in this case, $m = 4$
- Suppose we schedule the jobs using the **longest processing time first (LPT)** rule:
  - First, schedule the m longest jobs on the m machines
  - Whenever a machine becomes free, put the longest unprocessed job on that machine
- Idea: LPT puts shorter jobs towards the end of the schedule, where they can be used to balance the loads on each machine
- For our problem, this yields a schedule that looks like this:



  - This kind of diagram is known as a **Gantt chart**
- Therefore, the makespan for the LPT schedule is
- It turns out that the makespan of an LPT schedule is <u>always</u> at most $33.\overline{3}\%$ larger than the minimum makespan
- So... can we do better?
- Let's formulate this problem as a dynamic program

- Stages:

- States in stage t (nodes):

- Decisions, transitions, and rewards/costs at stage t (edges):

- Shortest/longest path?

- Minimum makespan $\longleftrightarrow$

- Assignments of jobs to machines $\longleftrightarrow$

A Problems

Problem 1 (Farmer Jones)**.** Farmer Jones decides to supplement the farm's income by baking and selling two types of cakes, pound cake and angel food cake. Each pound cake sold has a profit of \$3, while each angel food cake sold has a profit of \$4. Each pound cake uses 4 eggs and 4 cups of flour, while each angel food cake uses 6 eggs and 2 cups of flour. If Farmer Jones only has 48 eggs and 32 cups of flour available, how many of each type of cake should Farmer Jones bake in order to maximize profit? Assume all cakes baked are sold.

Formulate this problem as a dynamic program by giving its shortest/longest path representation.