

**Quiz 1 – 1/20/2022**

**Instructions.** You have 15 minutes to complete this quiz. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	3	
2	0.5	
3	0.5	
Total		/ 40

**Problem 1.** Fluttering Duck Airlines is starting operations at the small airport in Simplexville. The airline needs to purchase a new tractor to bring luggage to and from the airplanes. A new mechanized system will be installed in 3 years, so the tractor will not be needed after that. However, the tractor will receive heavy use, so the running and maintenance costs will increase rapidly after it ages. As a result, it may still be more economical to replace the tractor after 1 or 2 years. The total net cost of purchasing a tractor at the beginning of year  $i$  and trading it in at the beginning of year  $j$  is (in thousands of \$):

$i \downarrow j \rightarrow$	2	3	4
1	8	18	31
2		10	21
3			12

The goal is to determine what times (if any) the tractor should be replaced to minimize the total cost of having a tractor over the next 3 years.

Formulate this problem as a shortest path problem. In particular:

- draw the directed graph (nodes and edges),
- specify the edge lengths, and
- specify the source and sink nodes.

Suppose you solved the shortest path problem you formulated in Problem 1 with an algorithm that outputs (i) the length of a shortest path, and (ii) the nodes and edges in a shortest path.

**Problem 2.** Briefly explain how you would use this output to determine the minimum total cost of having a tractor over the next 3 years.

**Problem 3.** Briefly explain how you would use this output to determine when to purchase a new tractor. Give a hypothetical example if it helps.