Name: Feedback

SA367 · Mathematical Models for Decision Making — Spring 2022 · Uhan

Quiz 8 – 4/14/2022

Instructions. You have 15 minutes to complete this quiz. You may not use any other materials (e.g., notes, homework, website).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1a	0.5	
1b	0.5	
2	2	
Total		/ 30

Start with Problem 2 on the reverse side.

Problem 1. Solving for the value-to-go function $f_t(n)$ and the optimal decisions x_t^* of the stochastic dynamic program defined in Problem 2, we obtain:

t	n	$f_t(n)$	x_t^*	t	n	$f_t(n)$	x_t^*
1	0	0	0	3	0	0	0
1	1	47200	1	3	1	120000	1
1	2	135800	1	3	2	250000	0
2	0	0	0	4	0	0	-
2	1	68000	1	4	1	250000	-
2	2	187500	2	4	2	250000	-

a. Based on the table above, what is the company's minimum total expected cost?

Most of you had the right idea here. Some of you added multiple value-to-go function values, which is incorrect. Note that on page 2, the value-to-go function is defined as

$f_t(n)$ = minimum total expected cost for production runs $t, \dots, 3$ with n acceptable chips still needed — for $t = 1, 2, 3, 4$; $n = 0, 1, 2$

In other words, the value-to-go function $f_t(n)$ is the entire expected cost from stage t to the end of the decision making process.

b. Based on the table above, if the company still needs 2 microchips by the time it reaches its 3rd production run, what should the company do?

Almost all of you had the right idea here. Note that this problem is asking about stage $t = 3$ and state $n = 2$. What does the above table tell you about this stage-state combination?

Problem 2. Strogatz Semiconductors has received an order to supply 2 custom microchips. These custom chips are difficult to produce, and the company may have to manufacture more than one chip to obtain one that is acceptable. The company has time to make no more than 3 production runs, and at most 2 chips can be produced in each run. The probability of having a certain number of acceptable chips in a given run depends on how many chips are produced:

	Probability of acceptable chips		
Number of chips produced	0	1	2
0	1	0	0
1	0.40	0.60	0
2	0.35	0.50	0.15

Each chip costs \$20,000 to produce, and excess chips are worthless. If 2 acceptable chips have not been obtained by the end of the third production run, the company is in breach of contract and must pay a penalty of \$250,000.

The objective is to determine how many chips to produce in each production run in order to minimize the total expected cost. Here is the start of a stochastic dynamic program that models this problem:

- Stages:

$$\begin{aligned} t = 1, 2, 3 \quad &\leftrightarrow \quad \text{production run } t \\ t = 4 \quad &\leftrightarrow \quad \text{end of process} \end{aligned}$$

- States:

$$n = \text{number of acceptable chips still needed} \qquad \text{for } n = 0, 1, 2$$

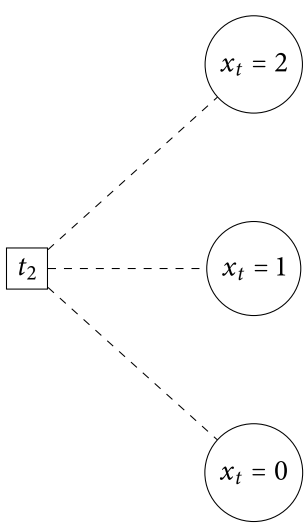
- Allowable decisions x_t at stage t and state n:

$$x_t \in \{0, 1, 2\} \quad \leftrightarrow \quad \text{number of chips to produce in production run } t \qquad \text{for } t = 1, 2, 3;\ n = 0, 1, 2$$

- Value-to-go function:

$$f_t(n) = \text{minimum total expected cost for production runs } t, \dots, 3 \text{ with } n \text{ acceptable chips still needed} \qquad \text{for } t = 1, 2, 3, 4;\ n = 0, 1, 2$$

Specify the transition probabilities and contributions from state $n = 2$ at stage t ($t = 1, 2, 3$) on the diagram below:



Many of you had the right idea here. If you struggled with this problem:

- Be careful with drawing the edges! Consider the following example. In the given diagram, we need $n = 2$ acceptable chips in stage t. If we decide to produce 1 chip in stage t, i.e, $x_t = 1$, then we might end up with 0 acceptable chips. In this case, we need $\underline{n = 2}$ acceptable chips in stage $t + 1$.
- Make sure you're using the correct probabilities from the table above.