Exam 1 – Information and Review Problems

1   Information

- When: **Thursday September 19** in class

- What: Lessons 1 – 7

- Calculators allowed

- No other outside materials allowed

- Review on Tuesday September 17 in class

- EI on Wednesday September 18, 1900 – 2030, CH348

2   Review Problems

Problem 1. Suppose Y is a random variable with cdf

$$F_Y(a) = \begin{cases} 0 & \text{if } a < 1, \\ 0.2 & \text{if } 1 \le a < 3, \\ 0.5 & \text{if } 3 \le a < 5, \\ 0.6 & \text{if } 5 \le a < 7, \\ 0.9 & \text{if } 7 \le a < 9, \\ 1 & \text{if } a \ge 9 \end{cases}$$

a. Is Y discrete or continuous? Briefly explain why.
b. What is the pmf/pdf of Y?
c. What is the expected value of Y?
d. What is the variance of Y?
e. What is the maximum possible value of Y? Briefly explain why.

Problem 2. Suppose X is a continuous random variable with pdf

$$f_X(a) = \begin{cases} 0 & \text{if } a < 0, \\ a & \text{if } 0 \le a \le 1, \\ 2 - a & \text{if } 1 < a \le 2, \\ 0 & \text{if } a > 2 \end{cases}$$

a. What is the cdf of X?
b. What is the expected value of X?
c. What is the variance of X?
d. What is the probability that X is in the interval $[1/2, 3/4]$?
e. What is the maximum possible value of X? Briefly explain why.

Problem 3. A poker hand of five cards is dealt from a normal deck of 52 cards. Let X be the number of aces in the hand.

    a. What is the probability that the hand contains more than one ace, given that it contains at least one ace?

    b. What is the probability that the hand contains more than one ace, given that it contains the ace of spades?

    c. Compare your answers from parts a and b. Is one probability larger or smaller than the other? Does this make sense intuitively? Briefly explain why.

Problem 4. A nickel is tossed 5 times in succession. Every time that the nickel comes up heads, a dime is tossed. Let X count the number of heads appearing on tosses of the dime.

    a. What is the probability that $X = 0$?

    b. What is the expected value of X?

Problem 5. Suppose `random()` is a function that can output random variates of a uniformly distributed random variable on $[0, 1]$. Give an algorithm that outputs random variates for the random variable X described in Problem 2.

Problem 6 (Nelson 2.9, 4.5). The following design is proposed for an automated manufacturing cell: A new job will arrive at the cell at precisely 30 minute intervals, and jobs will be processed first-come-first-served. The jobs can be classified into six types, depending upon the amount of time required to complete the job. Not all jobs can be completed in 30 minutes. It is believed that each arriving job is equally likely to be of each type. Formulate a stochastic process model for this system, including defining the random variables and the system-event functions.