SA402 – Dynamic and Stochastic Models — Fall 2013
Asst. Prof. Nelson Uhan

Final Exam – Information and Review Problems

1 Basic Information

- When: **Wednesday December 11**, 1330–1630
- Where: MI112 (Section 3041), MI115 (Section 5041)
- One 8.5 in × 11 in sheet of handwritten notes allowed
- Calculators allowed
- EI on Tuesday December 10, 1900 – 2030, CH348

2 Format

Part I Lessons 1 and 5–25 (everything except probability review)

- ~ 90 minutes
- ≥ 50% on Markov processes and queueing processes (Lessons 18-23)
- Your grade on this part will be your Final Exam grade

Part II Redemption for Exam 1 – Lessons 1–7

- Optional
- 2 problems, each worth 50% of the points you missed on Exam 1
- Your grade on this part will be added to your original Exam 1 grade
- No partial credit

Part III Redemption for Exam 2 – Lessons 8–17

- Same as Part II, except this part is for Exam 2

- Rationale:
  - If you understood the material during the semester and proved it on a previous exam, that's great!
  - If you understand the material by the end of the semester, that's great too!
    - But you have to show that you really understand it

3 Review Problems

These review problems cover the material in Lessons 18–23. Take a look at the review problems for Exam 1 and Exam 2 as well.

Problem 1. At the Markov and Co. management consulting firm, a consultant position can be occupied by an employee at any of three ranks: junior associate, senior associate, and partner. A junior associate becomes a senior associate after an exponentially distributed time with a mean of 3 years. A senior associate either gets promoted to partner after an exponentially distributed time with a mean of 6 years, or leaves the position after an exponentially distributed time with a mean of 4 years. Finally, a partner stays at that rank for an exponentially distributed time with a mean of 30 years, at which point he or she is replaced by a junior associate.

a. Model the movement between consultant ranks at Markov and Co. as a Markov process by defining
   - the state space and what each state means, and
   - the generator matrix.
b. What is the long-run fraction of consultants that are junior associates? Senior associates? Partners?

Problem 2. Customers arrive at Fantastic Dan's hair salon according to a Poisson process at a rate of 5 customers per hour. There are only 3 chairs provided for customers to wait for Dan. The time for Fantastic Dan to cut one customer's hair is exponentially distributed with a mean of 10 minutes. Since Fantastic Dan's customers are rather impatient, they may renege: the time a customer is willing to spend waiting before starting his or her haircut is exponentially distributed with a mean of 15 minutes.

a. Model this queueing system as a birth-death process by defining
   - the state space and what each state means,
   - the arrival rate in each state, and
   - the service rate in each state.
b. Over the long run, what is the expected number of customers in the hair salon?
c. Over the long run, what is the expected time a customer spends in the hair salon?

Problem 3. The Simplexville Community Center has 5 tennis courts. Pairs of players arrive at the courts according to a Poisson process with rate of one pair per 10 minutes, and use a court for an exponentially distributed time with mean 40 minutes.

a. What standard queueing model fits this setting best?
b. What is the traffic intensity in this queueing system?
c. What is the long-run expected fraction of time that all courts are empty?
d. Over the long run, how many pairs of players are waiting to get a court on average?
e. What is the long-run expected waiting time to get a court?
f. Suppose the playing times are actually uniformly distributed between 30 and 50 minutes. Give an approximation of the expected waiting time to get a court.

Problem 4. You have been put in charge of redesigning Turingtown's emergency call center. Based on historical data, you have estimated that calls arrive at a rate of 12 per hour, and that each call takes 1 operator 15 minutes on average. If all operators are busy when a call arrives, that call is put on hold. Model the call center as an $M/M/\infty$ queue and determine the minimum number of operators needed to ensure that no calls are put on hold 99% of the time in the long run.