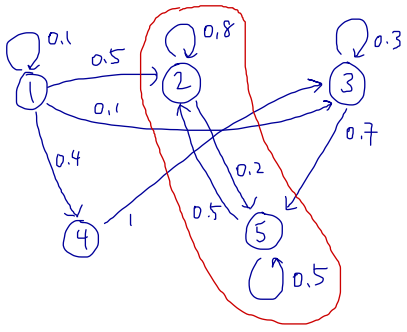
5 If we have time... (if not, finish for homework)

Example 5 (Nelson 6.6b, modified)**.**

a. Classify as recurrent or transient the states of the Markov chain with state space $\{1, 2, 3, 4, 5\}$ and the one-step transition matrix below by first finding all of the irreducible subsets of states.

$$\mathbf{P} = \begin{pmatrix} 0.1 & 0.5 & 0.1 & 0.1 & 0.2 \\ 0.0 & 0.8 & 0.0 & 0.0 & 0.2 \\ 0.0 & 0.0 & 0.3 & 0.0 & 0.7 \\ 0.0 & 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.5 & 0.0 & 0.0 & 0.5 \end{pmatrix}$$

b. For each irreducible set of states, find the steady-state probabilities.



transient: $\{1, 3, 4\}$

recurrent: irreducible set $\{2, 5\}$

Note that $\{2, 3, 5\}$ forms a self-contained Markov chain, but is not irreducible, since $\{2, 5\}$ is a subset that also forms a Markov chain.

$$\mathbb{P}_{RR} = \begin{pmatrix} 0.8 & 0.2 \\ 0.5 & 0.5 \end{pmatrix}$$

$$\mathbb{I} - \mathbb{P}_{RR} = \begin{pmatrix} -0.2 & 0.2 \\ 0.5 & -0.5 \end{pmatrix}$$

Steady-state prob. system of equations:

$$\left.\begin{aligned} -0.2\pi_2 + 0.5\pi_5 &= 0 \\ 0.2\pi_2 - 0.5\pi_5 &= 0 \\ \pi_2 + \pi_5 &= 1 \end{aligned}\right\} \Rightarrow \begin{aligned} \pi_2 &\approx 0.714 \\ \pi_3 &\approx 0.286 \end{aligned}$$