SA402 – Dynamic and Stochastic Models Fall 2013

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Lesson 7. A General Stochastic Process Model

1 A general stochastic process model

- Let's generalize the Bit Bucket example from last time
- Notation: $\mathbf{S}_n = \begin{pmatrix} S_{1,n} \\ \vdots \\ S_{m,n} \end{pmatrix}$ is a vector of m random variables
- $\{\mathbf{S}_n; n = 0, 1, 2, \ldots\}$ is the **state-change process**
  - Represents all relevant information about system status
- $\{T_n; n = 0, 1, 2, \ldots\}$ is the **event-epoch process**
  - T_n is the time of the nth system event
- $\{\mathbf{Y}_t; t \geq 0\}$ is the **output process**, defined by $\mathbf{Y}_t \leftarrow \mathbf{S}_n$ for $t \in [T_n, T_{n+1})$
  - Connects state-changes with times that they occur
- **Simulation algorithm**
  - System events $e_1, e_2, \ldots, e_k$
    - Update the new system state $\mathbf{S}_{n+1}$ from previous system state $\mathbf{S}_n$
    - Reset clocks $\mathbf{C}$ if necessary
  - Initial system event e_0
  - Clocks $\mathbf{C} = (C_1, C_2, \ldots, C_k)$

algorithm Simulation:

1:	$n \leftarrow 0$	(initialize system event counter)
	$T_0 \leftarrow 0$	(initialize event epoch)
	$e_0()$	(execute initial system event)
2:	$T_{n+1} \leftarrow \min\{C_1, \ldots, C_k\}$	(advance time to next pending system event)
	$I \leftarrow \arg\min\{C_1, \ldots, C_k\}$	(find index of next system event)
3:	$\mathbf{S}_{n+1} \leftarrow \mathbf{S}_n$	(temporarily maintain previous state)
	$C_I \leftarrow \infty$	(event I no longer pending)
4:	$e_I()$	(execute system event I)
	$n \leftarrow n + 1$	(update event counter)
5:	go to line 2	

- Comments:
  - $\mathbf{S}_{n+1} \leftarrow \mathbf{S}_n$ in Step 3 is for convenience
    - With this, system-event functions only need to specify changes in system state
  - By construction, $(\mathbf{S}_{n+1}, \mathbf{C})$ only depends on $(\mathbf{S}_n, \mathbf{C})$
    - Don't need complete history of states, just the previous state
    - We will exploit this later
- A stochastic process is a model describing a collection of time-ordered random variables that represent possible sample paths
- A sample path is a collection of time-ordered data describing how a process actually did behave in one instance

2 The Case of Copy Enlargement, revisited

The Darker Image, a national chain of small photocopying shops, currently configures each store with one photocopying machine and one clerk. Arriving customers stand in a single line to wait for the clerk. The clerk completes the customers' photocopying jobs one at a time, first-come-first-served, including collecting payment for the job.

- Let's formulate a stochastic process model for the copy shop as it currently operates
- Assumptions:
  - Interarrival times are independent with common cdf F_G
  - Service times are independent with common cdf F_X
  - Interarrival times and service times are independent
- System events:
- System state:

- System event algorithms:

- Output process:

- Time-average number of customers waiting for service over the first 6 hours:

- Time-average number of copiers in use – the **utilization** of the copier – over the first 6 hours:

- In words, what is $\int_0^6 Y_{1,t}\,dt$?