What if every transition is independent of the previous state?

Suppose S_{n+1} – the state at time step $n+1$ – is independent of $S_0, S_1, \ldots, S_n$ – the entire history of states up to and including time step n:

$$\Pr\{S_{n+1} = j \,|\, S_n = i, S_{n-1} = a, \ldots, S_0 = z\} = \Pr\{S_{n+1} = j\}. \tag{1}$$

Note that in this case, S_{n+1} does not even depend on S_n.

Since we assumed that S_{n+1} and S_n are independent,

$$\Pr\{S_{n+1} = j\} = \Pr\{S_{n+1} = j \,|\, S_n = i\}. \tag{2}$$

As a result, the Markov property still holds, since equations (1) and (2) imply:

$$\Pr\{S_{n+1} = j \,|\, S_n = i, S_{n-1} = a, \ldots, S_0 = z\} = \Pr\{S_{n+1} = j \,|\, S_n = i\}.$$

In words, the Markov property says that the only thing that S_{n+1} can possibly depend on is S_n. So if S_{n+1} doesn't depend on anything, not even S_n, the Markov property is still satisfied.