SA402 – Dynamic and Stochastic Models — Fall 2013

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Lesson 17. Markov Chains – Modeling and Assumptions

- Algorithmic representation of a Markov chain:
  - State space $\mathcal{M} = \{1, \dots, m\}$
  - One-step transition probabilities p_{ij} represented by cdfs $F_{N \mid L=i}(j)$ for all $i, j \in \mathcal{M}$
  - Initial state probabilities p_j represented by cdf F_{S_0}
  - S_n = state at time step n

$e_0()$: (generate initial state)

1: $S_0 \leftarrow F_{S_0}^{-1}(\texttt{random()})$ (initialize state of the process)

$e_1()$: (go to next state)

1: $S_{n+1} \leftarrow F_{N \mid L=S_n}^{-1}(\texttt{random()})$ (next state depends on current state)

algorithm Simulation:

1: $n \leftarrow 0$ (initialize system-event counter)
   $e_0()$ (execute initial system event)

2: $e_1()$ (update state of the system)
   $n \leftarrow n + 1$ (update system-event counter)

3: go to line 2

- Two assumptions:
  1. The **Markov property**: only the last state influences the next state

$$\Pr\{S_{n+1} = j \mid S_n = i, S_{n-1} = a, \dots, S_0 = z\} = \Pr\{S_{n+1} = j \mid S_n = i\}$$

  2. The **time stationarity property**: one-step transition probabilities don't depend on when the transition happens

$$\Pr\{S_{n+1} = j \mid S_n = i\} \text{ is the same for all } n = 0, 1, 2, \dots$$

- Last two lessons: time-dependent and time-independent performance measures
  - What is the probability that the process is in a certain state in n steps? In the long-run?
- Today: when is a Markov chain an appropriate model?

Example 1. For each of the following 5 cases, discuss what assumptions need to be made in order for the Markov property and time stationarity property hold, and whether these assumptions are plausible.

Case 1. A model of the movement of a taxi defines the state of the system to be the region of the city that is the destination of the current ride, and the time index to be the number of riders the taxi has transported. When the taxi delivers a rider, it stays in the destination region until it picks up another rider.

Case 2. A model of the weather in Annapolis defines the state of the system to be the high temperature, in whole degrees, and the time index to be the number of days.

Case 3. A model of computer keyboard use defines the state of the system to be the key that a person is currently typing, and the time index to be the number of keys typed.

Case 4. A model of the preferences of consumers for brands of toothpaste defines the state of the system to be the brand of toothpaste the consumer currently uses, and the time index to be the number of tubes of toothpaste purchased.

Case 5. A model of an industrial robot defines the state of the system to be the task that the robot is performing, and the time index to be the number of tasks performed. The robot works on two different kids of assemblies, each with its own distinct collection of tasks, and it requires a different tool for each kind of assembly. Changing the tool is one of the robot's tasks.

Example 2. Define a Markov chain model for one of the cases above by: (1) specifying the state space in detail, (2) specifying the meaning of the nth state in the case's context, (3) describing the meaning of the one-step transition probabilities and initial state probabilities in the case's context.

Example 3. The actuarial group at Arrow Auto Insurance wants to model the likelihood that a policyholder will be in an accident in a given year. The insurance company believes that after having an accident, a policyholder is a more careful driver for the next 2 years. Can this be modeled as a Markov chain? Why or why not?