Example 3 (Nelson 7.5, modified)**.** The Football State University motor pool maintains a fleet of vans to be used by faculty and students for travel to conferences, field trips, etc. Requests to use a van occur at about 8 per week on average (i.e. 8/7 per day), and a van is used for an average of 2 days. If someone requests a van and one is not available, then the request is denied and other transportation, not provided by the motor pool, must be found. The motor pool currently has 4 vans, but due to university restructuring, it has been asked to reduce its fleet. In order to argue against the proposal, the director of the motor pool would like to predict how many requests for the vans will be denied if the fleet is reduced from 4 to 3.

a. Model the 3-van system as a Markov process.
b. In the long run, what is the rate at which requests are denied?
c. In the long run, what is the average number of vans in use?

a. State space: $\mathcal{M} = \{0, 1, 2, 3\}$ ← #vans in use

Transition rates:

$$\mathcal{G} = \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} -8/7 & 8/7 & 0 & 0 \\ 1/2 & -23/14 & 8/7 & 0 \\ 0 & 1 & -15/7 & 8/7 \\ 0 & 0 & 3/2 & -3/2 \end{bmatrix} \end{array} \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array}$$

Steady-state eqns:

$$-\tfrac{8}{7}\pi_0 + \tfrac{1}{2}\pi_1 = 0$$
$$\tfrac{8}{7}\pi_0 - \tfrac{23}{14}\pi_1 + \pi_2 = 0$$
$$\tfrac{8}{7}\pi_1 - \tfrac{15}{7}\pi_2 + \tfrac{3}{2}\pi_3 = 0$$
$$\tfrac{8}{7}\pi_2 - \tfrac{3}{2}\pi_3 = 0$$
$$\pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$$

$$\Rightarrow \pi_0 \approx 0.127 \quad \pi_2 \approx 0.331$$
$$\pi_1 \approx 0.290 \quad \pi_3 \approx 0.252$$

b. Long-run denial rate $= \left(\frac{8}{7} \text{ requests/day}\right)(\pi_3) \approx 0.288$ requests/day

c. Long-run average number of vans in use $= 0\pi_0 + 1\pi_1 + 2\pi_2 + 3\pi_3 \approx 1.708$