Name: Feedback

SA402 · Dynamic and Stochastic Models — Fall 2022 · Uhan

Exam 0 – 9/16/2022

Instructions

- You have 50 minutes to complete this exam.
- You may use your plebe-issue TI-36X Pro calculator.
- You may not use any other materials.
- **No collaboration allowed.** All work must be your own.
- **Show all your work.** To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.
- Keep this booklet intact.
- **Do not discuss the contents of this exam with any midshipmen until it is returned to you.**

Problem	Weight	Score
1a	1	
1b	1	
1c	1	
1d	1	
2a	1	
2b	1	
2c	1	
2d	1	
3a	1	
3b	1	
3c	1	
4a	1	
4b	1	
5a	1	
5b	1	
Total		/ 150

Problem 0. Copy and sign the honor statement below. This exam will not be graded without a signed honor statement.

The Naval Service I am a part of is bound by honor and integrity. I will not compromise our values by giving or receiving unauthorized help on this exam.

Problem 1. Consider the random variable X with the following pdf:

$$f_X(a) = \begin{cases} 0 & \text{if } a < -1, \\ \frac{3}{2}a^2 & \text{if } -1 \leq a < 1, \\ 0 & \text{if } a \geq 1. \end{cases}$$

a. What is the cdf of X? Make sure to define the cdf $F_X(b)$ for all values of b between $-\infty$ and $+\infty$.

See Example 4 in Lesson 2 and Problem 2 in the Review Problems for Exam 0 for similar examples.

In addition, when defining the cdf, be careful with the input variable. For example, $F_X(b)$ should depend on the input variable b (and not some other variable, like a).

b. What is the expected value of X?

See Problem 2 in the Exercises for Lesson 2 and Problem 2 in the Review Problems for Exam 0 for similar examples.

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c. What is the minimum value that the random variable X can output? Briefly explain.

See Problem 2 in the Exercises for Lesson 2 for a similar example.

d. For the random variable X, which is more likely: a value near -1, or a value near 0? Briefly explain.

See Example 2 in Lesson 2 for a similar example.

Problem 2. Consider the random variable Y with the following cdf:

$$F_Y(a) = \begin{cases} 0 & \text{if } a < -3, \\ 0.2 & \text{if } -3 \leq a < 1, \\ 0.7 & \text{if } 1 \leq a < 7, \\ 1 & \text{if } a \geq 7. \end{cases}$$

a. What is the pmf of Y?

See Problem 1 in the Exercises for Lesson 2 and Problem 1 in the Review Problems for Exam 0 for similar examples.

Many of you used f_Y to denote the pmf. You were not penalized for this, but remember our convention for this class: p_Y denotes the pmf of a random variable Y.

Similar to Problem 1, when defining the pmf, be careful with the input variable. For example, $p_X(a)$ should depend on the input variable a (and not some other variable, like x).

b. What is the probability that $0 < Y \leq 4$?

Recall that $\Pr\{a < Y \leq b\} = F_Y(b) - F_Y(a)$ for any random variable Y, discrete or continuous.

c. What is the probability that $Y = 5$?

For this problem, first think about which values Y can output.

d. What is the variance of Y?

See Problem 1 in the Exercises for Lesson 2 and Problem 1 in the Review Problems for Exam 0 for similar examples.

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Problem 3. Simplex Pizza sells New York style and Sicilian style pizza by the slice. Let N represent the number of New York style slices in one order, and let T represent the total number of slices in one order. The joint pmf p_{NT} for N and T is:

	p_{NT}	T = 1	2	3
N	0	0.10	0.05	0.01
	1	0.25	0.10	0.02
	2	0	0.35	0.04
	3	0	0	0.08

a. What is the probability that an order contains a total of 2 slices?

See Example 1 in Lesson 3 for a similar example.

b. Explain why $p_{NT}(2,1) = p_{NT}(3,1) = p_{NT}(3,2) = 0$.

All of you had the right idea here.

c. What is the probability that an order contains 2 New York style slices, given that the order contains a total of 2 slices?

See Example 3 in Lesson 3 for a similar example.

Problem 4. As an analyst for Simplex Pizza, you have determined that the delivery times (in hours) are best modeled using a random variable Z with the following cdf:

$$F_Z(a) = \begin{cases} 0 & \text{if } a < 0, \\ 1 - e^{-4a} & \text{if } a \geq 0. \end{cases}$$

The company promises delivery within 0.5 hours or the pizza is free.

a. What is the probability that a delivery takes more than 0.5 hours?

Recall that $F_Z(a) = \Pr\{Z \leq a\}$.

b. What is the probability that a delivery takes more than 0.5 hours, given that a customer has already waited 0.25 hours?

See Example 7 in Lesson 3 for a similar example.

A number of you identified that the memoryless property of the exponential distribution could be used here, but unfortunately, applied it incorrectly.

Roughly speaking, if an exponential random variable Z is used to model the time we need to wait until the arrival of a customer, the memoryless property means that it does not matter how long we have waited so far. More specifically, if we haven't seen a customer by time a, then the time we need to wait starting from time a is distributed the same as the time we need to wait starting at time 0. Formally, this translates to:

$$\Pr\{Z > z + a \mid Z > a\} = \Pr\{Z > z\}$$

Note that the memoryless property does not mean

$$\Pr\{Z > z + a \mid Z > a\} = \Pr\{Z > z + a\}$$

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Problem 5. The Orange Company was having problems with its automated manufacturing cells yesterday: sometimes a tablet came out of a cell defective. 50% of the tablets were produced in cell 1, 30% in cell 2, and 20% in cell 3. 2% of the tablets produced in cell 1 came out defective, 3% in cell 2, and 5% in cell 3.

Suppose you select 1 tablet made yesterday at random. Let C be a random variable that represents the cell it was produced in (i.e., $C = 1$, 2 or 3). In addition, let D represent a random variable indicating whether the tablet came out defective (i.e., $D = 1$ if defective, 0 otherwise).

a. What is the probability that the randomly selected tablet came out defective, i.e. $\Pr\{D = 1\}$?

See Problem 3 in the Review Problems for Exam 0 for a similar example.

b. Are C and D independent? Give a numerical argument for why or why not.

Recall that X and Y are not independent if

$$\Pr\{X \in \mathcal{A} \text{ and } Y \in \mathcal{B}\} \neq \Pr\{X \in \mathcal{A}\} \Pr\{Y \in \mathcal{B}\} \quad \text{for some } \mathcal{A} \text{ and } \mathcal{B}$$

Another way to show that X and Y are not independent is to show that

$$\Pr\{Y \in \mathcal{B} \mid X \in \mathcal{A}\} \neq \Pr\{Y \in \mathcal{B}\} \quad \text{for some } \mathcal{A} \text{ and } \mathcal{B}$$

Most of you had the right approach here. Many of you, however, incorrectly identified the probabilities you used to show one of above.

Additional page for scratchwork or solutions