Solutions to Problem 1.

a. In this situation, time-stationary means that demand is not time-dependent (i.e., no seasonal demand), and the market for A, B, C is not time-dependent (i.e., not increasing or decreasing).

b. Let Y_t = total sales up to week t. Y_t follows a Poisson process with arrival rate 10 + 10 = 20.

$$\begin{aligned}\Pr\{Y_1 > 30\} &= 1 - \Pr\{Y_1 \le 30\}\\ &= 1 - \sum_{j=0}^{30} \frac{e^{-20(1)}(20(1))^j}{j!} \approx 0.013\end{aligned}$$

c. Let $Y_{A,t}$ = total sales of A up to week t, $Y_{B,t}$ = total sales of B up to week t, and $Y_{C,t}$ = total sales of C up to week t. $Y_{A,t}$ follows a Poisson process with arrival rate $20(0.2) = 4$, $Y_{B,t}$ follows a Poisson process with arrival rate $20(0.7) = 14$, and $Y_{C,t}$ follows a Poisson process with arrival rate $20(0.1) = 2$.

Expected sales in 1 month:

$$E[Y_{A,4}] = 4(4) = 16 \qquad E[Y_{B,4}] = 14(4) = 56 \qquad E[Y_{C,4}] = 2(4) = 8$$

Expected person-hours for 1 month:

$$25E[Y_{A,4}] + 15E[Y_{B,4}] + 40E[Y_{C,4}] = 25(16) + 15(56) + 40(8) = 1560$$

d. Let $Y^L_{B,t}$ = Louise's total sales of B up to week t. $Y^L_{B,t}$ follows a Poisson process with rate $10(0.7) = 7$.

$$\begin{aligned}\Pr\{Y^L_{B,2} - Y^L_{B,1} > 5 \text{ and } Y^L_{B,1} > 5\} &= \Pr\{Y^L_{B,2} - Y^L_{B,1} > 5\}\Pr\{Y^L_{B,1} > 5\} \quad \text{(independent increments)}\\ &= \Pr\{Y^L_{B,1} > 5\}\Pr\{Y^L_{B,1} > 5\} \quad \text{(stationary increments)}\\ &= \left(1 - \sum_{j=0}^{5} \frac{e^{-7(1)}(7(1))^j}{j!}\right)^2 \approx 0.4890\end{aligned}$$