

### Lesson 3. Performance Measures

- We simulate a system in order to understand how the system performs
- Some common **performance measures** in systems with queues:
  - average time a customer spends in the system
  - server utilization
  - time average number of customers

#### 1 Average time a customer spends in the system

- In this lesson's spreadsheet, we have the simulation of the first 7 customers at the bank drive-in window
  - Note that the random samples are actually not random (i.e., generated by RAND) – this is for illustrative purposes, so we all get the same results in class
- To find the average time that the first 7 customers spend at the drive-in window (including waiting in the queue), we can use the AVERAGE function

#### 2 Server utilization

- The **server utilization**  $\rho$  is the proportion of time that the server is busy serving a customer, or in other words

- What do you think are desirable values of  $\rho$ ?

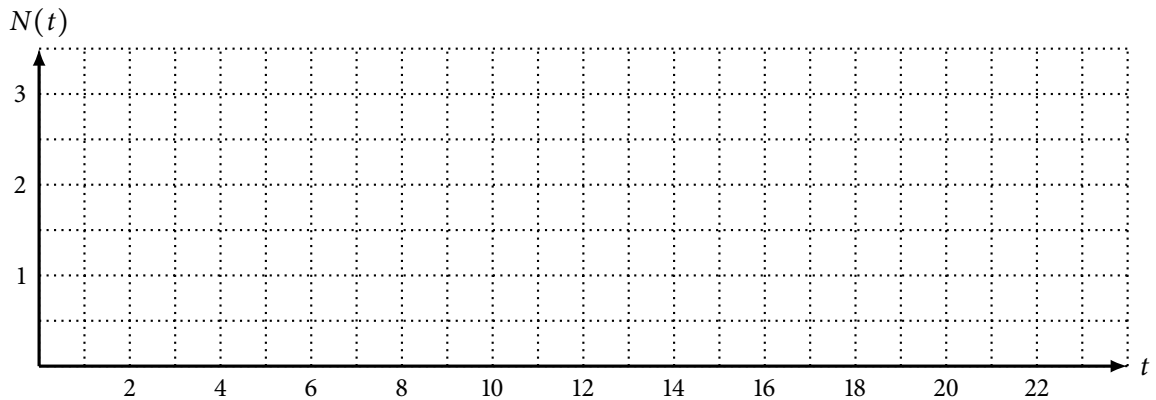
#### 3 Time average number of customers

- The time average number of customers  $L$  in time interval  $[0, T]$  is the average number of customers in the system at any time in  $[0, T]$ ; mathematically speaking:
  - Let  $N(t)$  be the number of customers in the system at time  $t$
  - Then

- How can we compute  $N(t)$ ?
  - e.g. Based on the bank drive-in window simulation in today's spreadsheet, what is  $N(0)$ ?  $N(2)$ ?  $N(6)$ ?  $N(18.7)$ ?  $N(20.1)$ ? (Don't look ahead or at the textbook!)

### 3.1 A straightforward way

- Let's compute  $N(t)$  for  $0 \leq t \leq 21$
- Arrange the events at the bank in chronological order
  - Copy and paste the events
  - Sort chronologically: highlight, then Data → Sort
- Use this timeline to keep track of how many customers are at the bank, and therefore compute  $N(t)$
- Let's graph  $N(t)$  for our example:



- Then the time average number of customers in time interval  $[0, 21]$  is

- This method is hard to automate in a spreadsheet...

### 3.2 Using a clever observation

- In the graph of  $N(t)$ , each customer contributes how long he or she has been in the system in the interval  $[0, T]$
- We can compute these contributions easily in the spreadsheet
- For example, for the 7th customer, an Excel formula is:

- Be careful to consider if the customer is in the system during the interval in question