SA421 – Simulation Modeling
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Lesson 4. Sample Mean, Sample Variance, Confidence Intervals

1 Overview

- Last time: computed and observed performance measures for 1 simulation run of the bank drive-in window
  - e.g. the average time that the first N customers spend at the bank
  - For simplicity, let's call this the "average bank time"
- The observed average bank time can differ between simulation runs
- The average bank time is a random variable
  - Uncertain quantity before the simulation run
  - Depends on interarrival times and service times, which are random variables
- Can we estimate the distribution of the average bank time?
  - Let's focus on estimating the mean and variance of this distribution

2 The experiment

- Run the simulation n times
- Compute performance measure (e.g. average bank time) for each simulation run
(obtaining n observations of the performance measure)
- Use the n observations to estimate the mean of the performance measure

3 After the experiment: observed sample mean and sample variance

- Let $X_1, \dots, X_n$ be independent and identically distributed (i.i.d.) random variables with unknown mean μ and variance σ^2
- Let $x_1, \dots, x_n$ be the observed values of $X_1, \dots, X_n$, respectively
  - Think of X_i as the average bank time in the ith simulation run before the experiment
  - Think of x_i as the observed average bank time in the ith simulation run after the experiment
  - Since the simulation runs replicate the same system, $X_1, \dots, X_n$ should be identically distributed
- We want to estimate μ
- The **observed sample mean** is

- The **observed sample variance** is

- The **standard error** is

- We estimate μ using the the observed sample mean
- We estimate σ^2 using the observed sample variance
- These are **point estimates** for μ and σ^2, respectively
- The standard error is a measure of the accuracy of the estimate of μ
- Why should we estimate μ and σ^2 this way?

4 Before the experiment: sample mean and sample variance

- The **sample mean** is

  - The sample mean $\overline{X}$ is a random variable: <u>before</u> the experiment, it is an uncertain quantity
  - $\mathbb{E}[\overline{X}] = \mu$, $\mathrm{Var}(\overline{X}) = \sigma^2/n$:

- The **sample variance** is

  - The sample variance is also a random variable: <u>before</u> the experiment, it is an uncertain quantity

- The sample mean is an **unbiased estimator** of μ, and the sample variance is an **unbiased estimator** of σ^2: that is,

  - Intuitively, this indicates that using the observed sample mean to estimate μ and the observed sample variance to estimate σ^2 is not a bad idea

5 How good is the observed sample mean as an estimate?

- Is the observed sample mean $\overline{x}$ "close" to μ?
- Suppose $\overline{X}$ is normally distributed
  - This is true if $X_1, \ldots, X_n$ are normally distributed
  - This is approximately true by the Central Limit Theorem if $n \geq 30$
- Then the $(1 - \alpha)100\%$ **confidence interval for** μ is

  - This is an **interval estimate** for μ
  - $t_{\alpha/2,n-1}$ can be computed by Excel with $\mathsf{TINV}(\alpha, n-1)$
  - The t-distribution with $n - 1$ degrees of freedom ≈ standard Normal distribution when $n \geq 30$
- Interpretation of a confidence interval:
  - Sample mean $\overline{X}$ and sample standard deviation S^2 are random variables
  - Every experiment, we get different observed sample mean $\overline{x}$ and observed sample variance s^2
  - ⇒ Every experiment, we get a different confidence interval
  - After running the experiment many times, $(1 - \alpha)100\%$ of the resulting confidence intervals will contain the actual mean μ
  - We say that "we are $(1 - \alpha)100\%$ confident that the mean μ lies within the confidence interval"
  - Wrong interpretation: "The mean μ lies within the confidence interval with $(1 - \alpha)100\%$ probability"
- Smaller confidence interval ⇒ more accurate estimate of μ

Example 1. Suppose an estimate of μ within 0.1 was desired at a confidence level of 95%. We perform a "warm-up" experiment of $n = 30$ simulation runs to compute an observed sample variance s^2, which is found to be 3.2. How many simulations runs are needed to obtain this estimate of μ?