

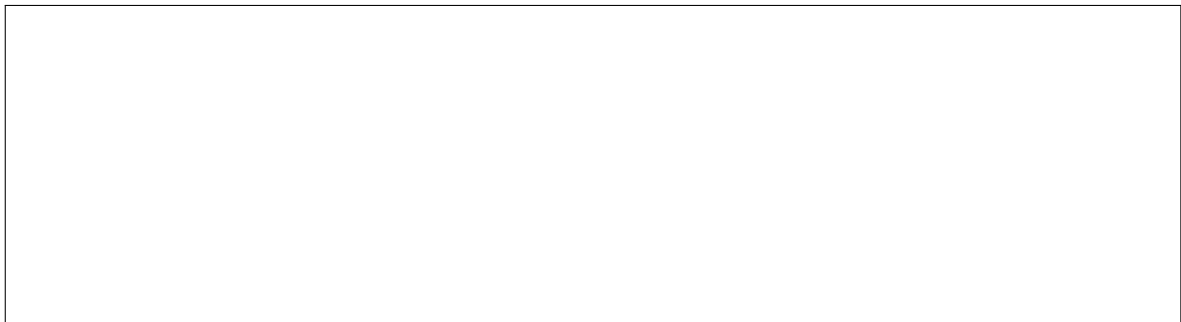
Lesson 11. Random Variate Generation and the Inverse Transform Method

1 Overview

- A **random variate** is a particular outcome of a random variable, e.g. interarrival time in the bank drive-in window, $\text{Uniform}[0, 1]$, $N(\mu, \sigma)$
- How can we generate random variates using a computer, especially when we have the cdf or pdf of the random variable?
- How can we test whether a set of values are generated from the desired distribution?

2 The Inverse Transform Method

- Let X be a continuous random variable with cdf F
- Let U be a $\text{Uniform}[0, 1]$ random variable
- **Key observation:** The random variable $Y = F^{-1}(U)$ has cdf F .



⇒ If we can generate values u from U , then we can generate values from X by plugging u into F^{-1}

- $Y = F^{-1}(U)$ is a **random variate generator** for X

Example 1. Let X be an exponential random variable with mean $\alpha = 1/\lambda$ (λ is called the rate parameter). The cdf of X is

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find a random variate generator for X .



Example 2. Let X be a Uniform $[a, b]$ random variable. The cdf of X is

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ 1 & \text{if } x > b \end{cases}$$

Find a random variate generator for X .

Example 3. Let X be a random variable with pdf

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ \frac{1}{2} & \text{if } 1 < x \leq 2 \\ 0 & \text{if } x > 2 \end{cases}$$

- a. Find the cdf of X . Recall that $F(x) = \mathbb{P}(X \leq x) = \int_{-\infty}^x f(x) dx$.
- b. Find a random variate generator for X .