SA421 – Simulation Modeling Spring 2013
Asst. Prof. Nelson Uhan

Lesson 11. Random Variate Generation and the Inverse Transform Method

1 Overview

- A **random variate** is a particular outcome of a random variable, e.g. interarrival time in the bank drive-in window, Uniform$[0,1]$, $N(\mu, \sigma)$
- How can we generate random variates using a computer, especially when we have the cdf or pdf of the random variable?
- How can we test whether a set of values are generated from the desired distribution?

2 The Inverse Transform Method

- Let X be a continuous random variable with cdf F
- Let U be a Uniform$[0,1]$ random variable
- **Key observation:** The random variable $Y = F^{-1}(U)$ has cdf F.

$\Rightarrow$ If we can generate values u from U, then we can generate values from X by plugging u into F^{-1}

- $Y = F^{-1}(U)$ is a **random variate generator** for X

Example 1. Let X be an exponential random variable with mean $\alpha = 1/\lambda$ (λ is called the rate parameter). The cdf of X is

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find a random variate generator for X.

Example 2. Let X be a Uniform$[a, b]$ random variable. The cdf of X is

$$F(x) = \begin{cases} 0 & \text{if } x < a \\ \frac{x-a}{b-a} & \text{if } a \le x \le b \\ 1 & \text{if } x > b \end{cases}$$

Find a random variate generator for X.

Example 3. Let X be a random variable with pdf

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } 0 \le x \le 1 \\ \frac{1}{2} & \text{if } 1 < x \le 2 \\ 0 & \text{if } x > 2 \end{cases}$$

a. Find the cdf of X. Recall that $F(x) = \mathbb{P}(X \le x) = \int_{-\infty}^{x} f(x)\,dx$.
b. Find a random variate generator for X.