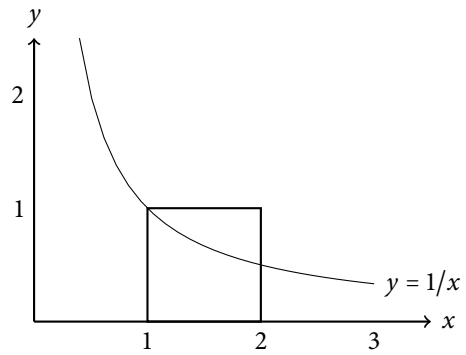
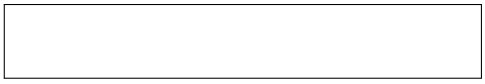
Lesson 14. Using Random Numbers to Estimate Area

- We can use random numbers and random variates to estimate areas and irrational numbers
- Recall that $\ln 2 = \int_1^2 \frac{1}{x}\,dx$
- We can estimate $\ln 2$ by estimating the area under the graph of $y = 1/x$ between 1 and 2



- Idea:
  - Throw n uniformly random points in the square $[1,2] \times [0,1]$
  - Suppose m of those points fall under the graph of $y = 1/x$
  - Then

- Recall that we can generate $V \sim \text{Uniform}[a,b]$ with the generator $V = a + (b-a)U$, where $U \sim \text{Uniform}[0,1]$
- How do we generate uniformly random points in the square $[1,2] \times [0,1]$?
  - Let U_1, U_2 be independent $\text{Uniform}[0,1]$ random variables
  - Let $X \sim \text{Uniform}[1,2]$, which can be generated by 
  - Let $Y \sim \text{Uniform}[0,1]$, which can be generated by 
  - (X, Y) is a uniformly random point in the square
  - The point (X, Y) is under the graph of $y = 1/x$ if $Y < 1/X$
- Let's use Excel to generate 30 uniformly random points in the square $[1,2] \times [0,1]$ and estimate $\ln 2$.

Problem 1. In this problem, we will use simulation to estimate π.

a. What is the area of a circle with radius 1?

b. Give an equation of the circle centered at the origin of the xy-plane with radius 1. Change your equation to an inequality that represents the interior of this circle.

c. Use Excel to generate 1000 uniformly random points in the square $[-1, 1] \times [-1, 1]$.

d. Use Excel to count how many of these points fall inside the circle centered at the origin with radius 1. Use this to give an estimate of π.