butcher

The Markov Butcher Shop sells premium grade A+ beef. Customers arrive at the butcher shop and form a single queue. There is one butcher who serves customers from the queue on a first-come-first-served basis.

Based on historical data, the interarrival time between customers is exponentially distributed with a mean of 8 minutes. The service times for customers is also exponentially distributed, but with a mean of 6 minutes. The interarrival times and service times are assumed to be independent.

a. Simulate by hand the service of the first 10 customers. Assume that you have an oracle that generates samples from the interarrival time and service time distributions: the first 10 samples of the interarrival time are 5, 5, 8, 3, 2, 5, 20, 2, 1, 3, and the first 10 samples of the service time are 4, 10, 6, 10, 14, 7, 10, 4, 15, 6.

b. What is the average waiting time of the first 10 customers?

c. What is the average delay of the first 10 customers?

d. What is the server utilization from the time the shop opens until the time the 10th customer departs the shop?

e. What is the time average number of customers in the shop, from the time the shop opens until the time the 10th customer departs the shop?

Solution

a. Using the same approach we used in class, we can simulate the system as follows:

| Customer	Interarrival time	Arrival time	Start time	Time in queue	Service time	Depart time	Time in system
1	5	5	5	0	4	9	4
2	5	10	10	0	10	20	10
3	8	18	20	2	6	26	8
4	3	21	26	5	10	36	15
5	2	23	36	13	14	50	27
6	5	28	50	22	7	57	29
7	20	48	57	9	10	67	19
8	2	50	67	17	4	71	21
9	1	51	71	20	15	86	35
10	3	54	86	32	6	92	38

b. The average waiting time of the first 10 customers can be found by taking the average of the time these customers spend in the system:

$$w = \frac{4 + 10 + 8 + 15 + 27 + 29 + 19 + 21 + 35 + 38}{10} = 20.6$$

c. The average delay of the first 10 customers can be found by taking the average of the time these customers spend in the queue:

$$w_q = \frac{0 + 0 + 2 + 5 + 13 + 22 + 9 + 17 + 20 + 32}{10} = 12$$

d. This is easy to compute once we graph of $N(t)$, the number of customers in the system at time t:
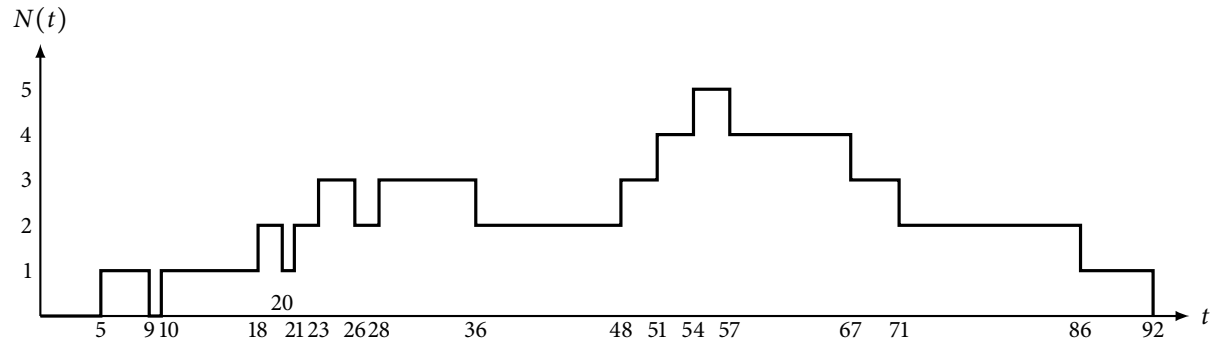
92 minutes elapses between the time the shop opens and the time the 10th customer departs the shop. As we can see, the butcher is idle for only 6 out of the 92 minutes. Therefore, the server utilization in this time interval is $\rho = 86/92 \approx 0.9347$.

e. From the graph of $N(t)$ above, the time average number of customers in the shop is

$$\ell = \frac{\begin{aligned}4(1) + 8(1) + 2(2) + 1(1) + 2(2) + 3(2) + 2(2) + 8(3) \\ + 12(2) + 3(3) + 3(4) + 3(5) + 10(4) + 4(4) + 15(2) + 6(1)\end{aligned}}{92} = \frac{207}{92} = 2.25$$