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Lesson 1. Introduction to Simulation

1 What is simulation?

- **Simulation** is the imitation of a real-world system in order to obtain data that can be used to evaluate and improve the system's performance
- In this course, we will focus on simulations that are
  - **stochastic**: some aspects of the system are modeled using random variables
  - **discrete-event**: state of the system changes at discrete points in time triggered by **events**, e.g. the arrival of a customer, the completion of an activity

2 Why simulate?

- Real-world trial-and-error approaches are expensive, time consuming and disruptive
- Complex systems are often resistant to analytical models and solutions
  (e.g. limitations of mathematical programming, queueing theory)

3 The Fantastic Dan Problem

Customers visit the neighborhood hair stylist Fantastic Dan for haircuts. The customer interarrival time is exponentially distributed with a mean of 20 minutes. Each haircut takes Fantastic Dan anywhere from 15 to 25 minutes, uniformly distributed.

- Questions we might be interested in:
  - On average, how long does a customer spend at the hair stylist?
  - On average, how many customers are waiting for a haircut?
- We can answer these questions by simulating the arrival and service of customers
- To perform this simulation, we need:
  - time of arrival of each customer
  - how long it takes to serve each customer
- Obtaining these requires **sampling** from the above probability distributions
- We will discuss the mathematics behind sampling later in the course
- For now, let's assume we have a sampling oracle for these distributions

3.1 Simulating the first 5 customers

- Fantastic Dan opens his shop at time 0
- The interarrival times of the first 5 customers are 23, 5, 10, 79, 13
- The service times of the first 5 customers are 17, 21, 19, 22, 19

Customer (a)	Interarrival time (b)	Arrival time (c)	Start service time (d)	Service time (e)	Departure time (f)	Total time in shop (g)

- Arrival time c = time when customer arrives at shop
- Start service time d = time when customer starts service
- Departure time f = time when customer leaves =
- Total time at shop g =
- Does arrival time c always equal start service time d?
- What assumptions are we making with these calculations?

4 Performance measures

- We simulate a system in order to understand how the system performs
- Some common **performance measures**, especially for systems with queues:
  - average waiting time and average delay
  - server utilization
  - time average number of customers

4.1 Average waiting time and average delay

- The **average waiting time** w is the average time a customer spends in the system from arrival to departure
- Offers a view of performance from the customer's perspective
- For the Fantastic Dan simulation above, the average waiting time is

- We can similarly compute the **average delay** w_q, the average time a customer spends in the queue

4.2 Server utilization

- The **server utilization** ρ is the proportion of time that the server is busy serving a customer:

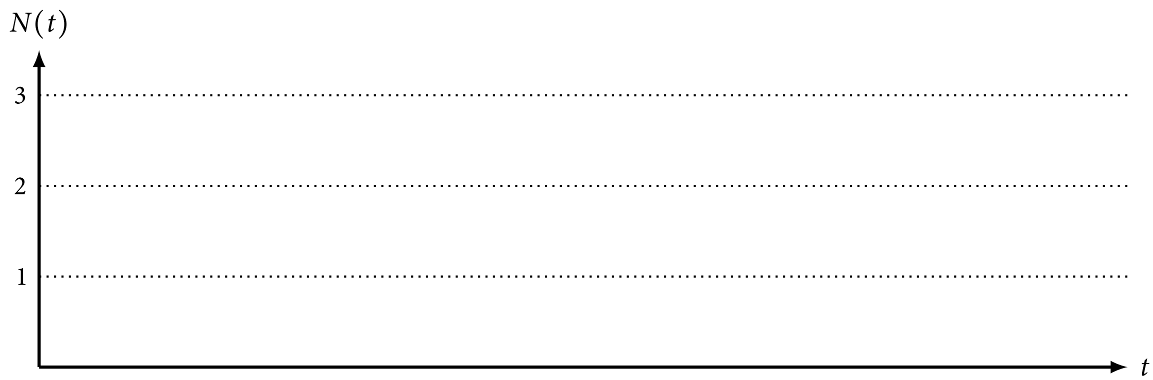
- Offers a view of performance from the system's perspective
- For the Fantastic Dan simulation above, the server utilization from the time the shop opens until the time the 5th customer departs is

4.3 Time average number of customers

- The **time average number of customers** ℓ in time interval $[0, T]$ is the average number of customers in the system at any time in $[0, T]$
- Offers another view of performance from the system's perspective
- Mathematically:
  - Let $N(t)$ be the number of customers in the system at time t

  $\Rightarrow$

- For the Fantastic Dan simulation above, let's graph $N(t)$ for $0 \leq t \leq 158$



- Then the time average number of customers in time interval $[0, 158]$ is
- Alternatively, suppose $N(t)$ changes value at $t_1, t_2, \ldots, t_{m-1}, t_m$
- For example, for the Fantastic Dan simulation above
- Then the time average number of customers can be computed as