SM223 – Calculus III with Optimization Fall 2012
Asst. Prof. Nelson Uhan

Exam 2 – Information and Review Problems

1 Information

- When: Wednesday October 3, in class
- What: Lessons 10 – 23
  - (The relevant parts of Sections 13.1 – 13.4, 14.1, 14.3, 14.4 in Stewart)
- No outside materials (e.g. notes, homework, books) allowed
- No calculators allowed
  - The exam will be designed so that only simple calculations are necessary
- Review on Monday October 1
  - We will discuss some of the problems below, as well as any questions that you might have
- Homework 24 (assigned on Friday September 28) is due on Thursday October 4
  - Due date is changed on WebAssign
- EI on Tuesday October 2, 1930 – 2130, CH348

2 Review Problems

Note: these problems together are not meant to represent the total length of the exam.

Problem 1. Let $\vec{r}(t) = (t)\vec{i} + (\cos \pi t)\vec{j} + (\sin \pi t)\vec{k}$.

a. Sketch the curve $\vec{r}(t)$, $t \geq 0$.
b. Find $\vec{r}'(t)$.
c. Find $\vec{r}''(t)$.
d. Find $\int_0^1 \vec{r}(t)dt$.
e. Find the unit tangent vector at the point where $t = 0$.

Problem 2. The positions of two airplanes at time t are given by the vector functions

$$\vec{r}_1(t) = \langle -3 + 4t, t^2, -6 + 5t \rangle \qquad \vec{r}_2(t) = \langle t^2, -12 + 7t, t^2 \rangle$$

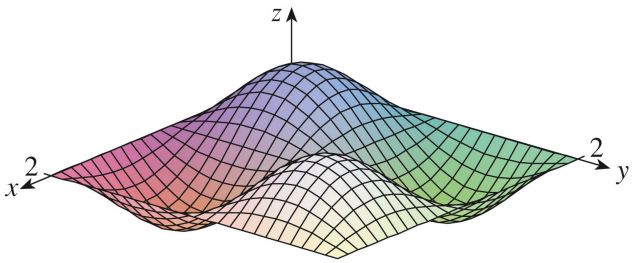
Do the airplanes collide? If not, do their paths intersect?

Problem 3. Find the length of the curve $\vec{r}(t) = \langle 2t, \cos 2t, \sin 2t \rangle$, $0 \leq t \leq 1$.

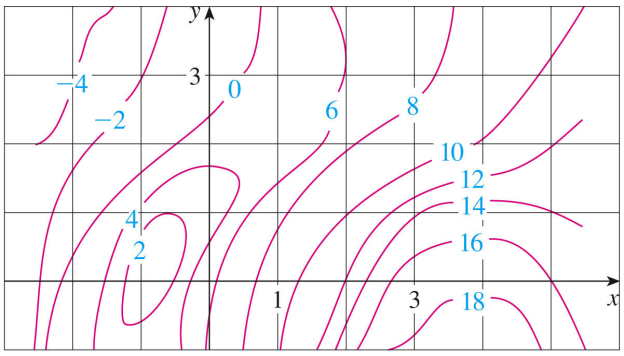
Problem 4. A ball is thrown from 2 m above the ground with an initial speed of 14 m/s and angle of elevation 60°. Use $g = 9.8$ m/s^2.

a. **Set up** an equation whose solution gives you the time t at which the ball lands. Do not solve the equation.
b. Suppose the solution to the equation you set up in part a is $t = -0.14, 1.38$. **Explain in words** how you would use this to obtain the range of the ball. You do not have to give a numerical value for the projectile's range.

Problem 5. Make a rough sketch of a contour map for the function whose graph is shown below.



Problem 6. A contour map for a function f is shown below.



a. Estimate $f(3,2)$.
b. Is $f_x(1,1)$ positive or negative?
c. Is $f_y(4,0)$ positive or negative?

Problem 7. Find the first partial derivatives.

a. $f(x,y) = \sqrt{2x + y^2}$
b. $f(x,y) = e^{-x}\sin 2y$
c. $g(u,v) = u \arctan v$
d. $f(x,y,z) = \dfrac{x}{y-z}$

Problem 8. Let $f(x,y) = e^x \cos y$.

a. Find $f_x(x,y)$.
b. Find $f_y(x,y)$.
c. Find $f_{xy}(x,y)$.
d. Find $f_{yx}(x,y)$.
e. Find an equation of the tangent plane to the surface $z = f(x,y)$ at $(0,0,1)$.
f. Use your answer in part e to approximate $f(0.1,-0.1)$.