SM223 – Calculus III with Optimization Fall 2012
Asst. Prof. Nelson Uhan

Exam 3 – Information and Review Problems

1 Information

- When: Monday October 29, in class
- What: Lessons 24 – 38 (Sections 14.5 – 14.8 in Stewart)
- You may bring one 3 in × 5 in index card of original, handwritten notes
- You may use a calculator to solve systems of equations
  - You may use a calculator to check your work for other tasks (e.g., arithmetic, algebra, derivatives), but you must show all work for these tasks to get full credit
- No other outside materials allowed
- Review on Friday October 26
  - We will discuss some of the problems below, as well as any questions that you might have
- EI on Sunday October 28, 1900 – 2100, CH348 (or nearby)

2 Review Problems

Note: these problems together are not meant to represent the total length of the exam.

Problem 1. Let $v = x^2 \sin y + ye^{xy}$, $x = s + 2t$, $y = st$. Use the chain rule to find $\partial v/\partial s$ and $\partial v/\partial t$ when $s = 0$ and $t = 1$.

Problem 2. A manufacturer has modeled its yearly production function P (the value of its entire production in millions of dollars) as the function

$$P(L, K) = 1.47L^{0.65}K^{0.35}$$

where L is the number of labor hours (in thousands) and K is the amount of invested capital (in millions of dollars). Suppose that when $L = 30$ and $K = 8$, the labor force is decreasing at a rate of 2000 labor hours per year, and capital is increasing at a rate of 500,000 dollars per year. Find the rate of change of production.

Problem 3. Let $f(x, y) = x^2 y + \sqrt{y}$.

a. Find $\nabla f(x, y)$.
b. i. Find the directional derivative of f at $(2, 1)$ in the direction towards the point $(5, 3)$.
   ii. Explain what this value means.
c. What is the maximum rate of change of f at $(2, 1)$?
d. In which direction does the maximum rate of change of f occur?

Problem 4. Find (a) an equation of the tangent plane and (b) parametric equations of the normal line to the surface $xy + yz + zx = 5$ at the point $(1, 2, 1)$.

Problem 5. Find the local maxima, local minima, and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

Problem 6. Find the extreme values of $f(x, y, z) = x + 2y$ subject to the constraints $x + y + z = 1$ and $y^2 + z^2 = 4$.

Problem 7. Find the extreme values of $f(x, y) = x^2 + y^2 + 4x - 4y$ on the region $C = \{(x, y) \mid x^2 + y^2 \leq 9\}$.

Problem 8. A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and girth (twice the sum of its width and height) is at most 108 in. Suppose you want to find dimensions of the package with the largest volume that can be mailed.

a. Write an optimization model for your problem (i.e. What is the function you are maximizing/minimizing? What are the constraints?)
b. Solve this optimization model using your method of choice.