SM223 – Calculus III with Optimization Fall 2012
Asst. Prof. Nelson Uhan

# Exam 4 – Information and Review Problems

## 1 Information

- When: Monday December 3, in class
- What: Lessons 41 – 53 (relevant parts of Sections 15.1–15.5, 15.7, 15.8 in Stewart)
- You may bring one 3 in × 5 in index card of original, handwritten notes
- Calculators are allowed
  - Emphasis will be on setting up double and triple integrals
- No other outside materials allowed
- Review on Friday November 30
  - We will discuss some of the problems below, as well as any questions that you might have
- EI on Sunday December 2, 1900 – 2100, CH348 (or nearby)

## 2 Review Problems

Note: these problems together are not meant to represent the total length of the exam.

**Problem 1.** Reverse the order of integration for each iterated integral.

a. $\int_{-1}^{1}\int_{2}^{3} e^{x^3+y}\,dy\,dx$

b. $\int_{0}^{1}\int_{x}^{1} \cos(y^2)\,dy\,dx$

c. $\int_{0}^{1}\int_{\sqrt{y}}^{1} \frac{ye^{x^2}}{x^3}\,dx\,dy$

**Problem 2.** Set up the following double and triple integrals as iterated integrals.

a. $\iint_D \frac{y}{1+x^2}\,dA$, where $D$ is bounded by $y = \sqrt{x}$, $y = 0$, and $x = 4$.

b. $\iint_D \frac{1}{1+x^2}\,dA$, where $D$ is the triangular region with vertices $(0,0)$, $(1,1)$, and $(0,1)$.

c. $\iint_D (x^2+y^2)^{3/2}\,dA$, where $D$ is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.

d. $\iint_D x\,dA$, where $D$ is the region in the first quadrant that lies between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$. Use polar coordinates.

e. $\iiint_D xy\,dV$, where $E$ is the solid that lies below that plane $z = x$ and above the triangular region with vertices $(0,0,0)$, $(\pi,0,0)$, and $(0,\pi,0)$.

f. $\iiint_D xy\,dV$, where $E$ is bounded by the paraboloid $x = 1 - y^2 - z^2$ and the plane $x = 0$.

g. $\iiint_D yz\,dV$, where $E$ lies above the plane $z = 0$, below the plane $z = y$, and inside the cylinder $x^2 + y^2 = 4$. Use cylindrical coordinates.

**Problem 3.** Set up two integrals to find the volume of the solid above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$: one in rectangular coordinates, and one in cylindrical coordinates.

**Problem 4.** Consider a lamina that occupies the region $D$ bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant with density function $\rho(x, y) = y$.

a. Set up an integral to find the mass of the lamina.
b. Set up integrals to find the center of mass of the lamina.

**Problem 5.** Set up integrals to find the mass and the center of mass of the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$ and density function $\rho(x, y, z) = x^2 + y^2 + z^2$.

**Problem 6.** Rewrite the iterated integral

$$\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} f(x, y, z)\,dz\,dy\,dx$$

using the following orders of integration:

a. $dz\,dx\,dy$
b. $dx\,dy\,dz$