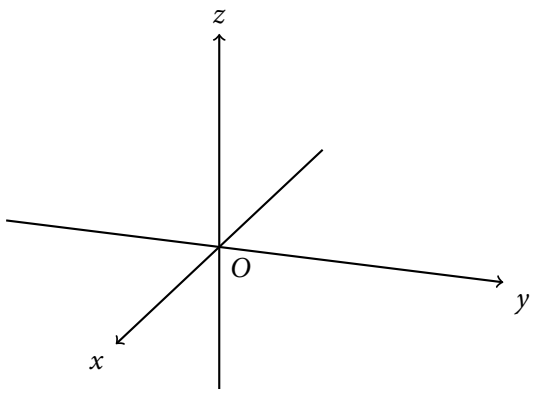
Lesson 1. 3D Coordinate Systems

1 Today...

- 3D rectangular coordinate system
- Distance formula in 3D
  - Equation for a sphere
- Graphing equations in 3D

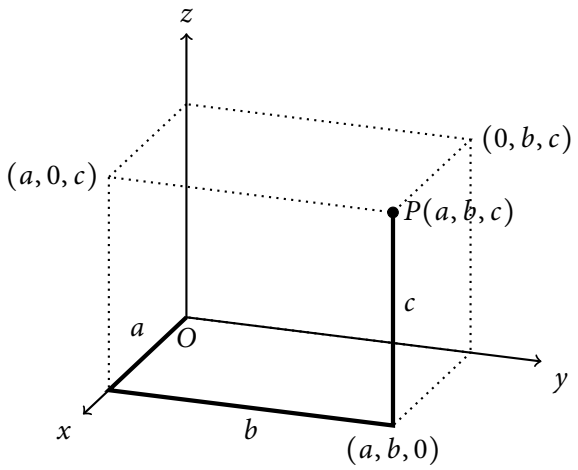
2 3D rectangular coordinate system

- How do we locate points in space?
- 3 mutually perpendicular **coordinate axes** through origin O:



- 3 **coordinate planes**
  - xy-plane contains the
  - xz-plane contains the
  - yz-plane contains the
- The coordinate planes divide space into 8 **octants**
  - The **first octant** is the octant with positive axes

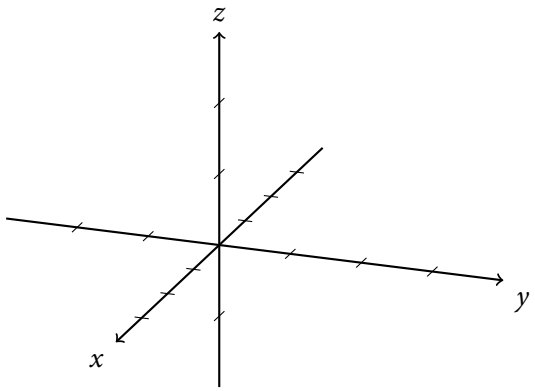
- Any point P in space can be represented an ordered triple (a, b, c):



- (a, b, c) are the **rectangular coordinates** of P (also known as **Cartesian coordinates**)
  - a is called the x-**coordinate** of P
  - b is called the y-**coordinate** of P
  - c is called the z-**coordinate** of P
- $(a, b, 0)$ is the **projection** of $P(a, b, c)$ on the
- $(0, b, c)$ is the projection of $P(a, b, c)$ on the
- $(a, 0, c)$ is the projection of $P(a, b, c)$ on the
- We often refer to three-dimensional space as $\mathbb{R}^3$

Example 1.

(a) Plot $P(3, -1, 2)$.



(b) What is the projection of P onto the xy-plane?

3 Distance formula in 3D

- The **distance** between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

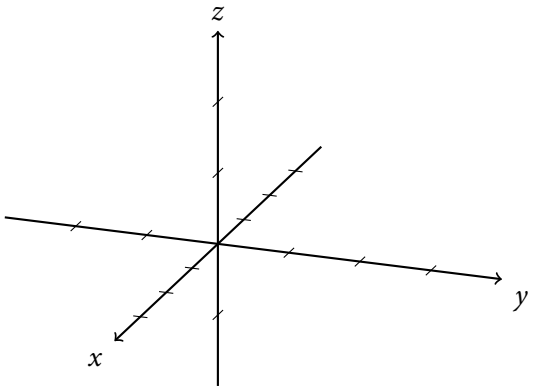
Example 2. What is the distance from the point $P(2, -1, 0)$ and $Q(4, 1, 1)$?

- A **sphere** is the set of all points $P(x, y, z)$ whose distance from a center $C(h, k, l)$ is radius r
  - The standard equation for a sphere with radius r and center (h, k, l) is

Example 3. Show that $x^2 + y^2 + z^2 - 2x - 4y + 8z = 15$ represents a sphere, and find its center and radius.

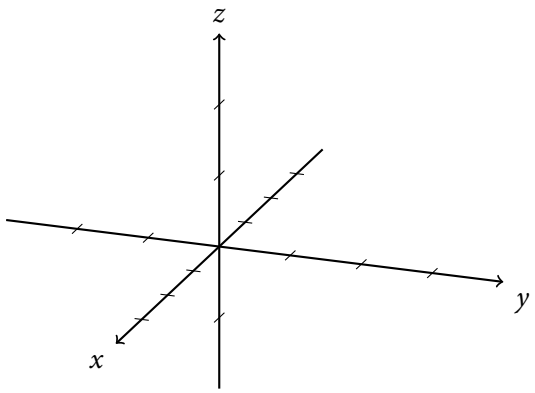
4 Graphing equations in 3D

Example 4. Which points (x, y, z) satisfy $y = 2$ in $\mathbb{R}^3$?

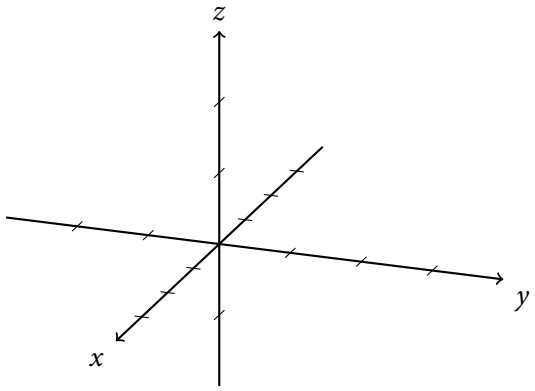


Example 5.

(a) Draw $y = x^2$ in $\mathbb{R}^3$.



(b) Draw $y = x^2, z = 2$ in $\mathbb{R}^3$.



Example 6. Give an equation for the

- xz-plane:
- xy-plane:
- yz-plane: