SM223 – Calculus III with Optimization Fall 2012
Asst. Prof. Nelson Uhan

Lesson 4. The Dot Product (cont.)

1 Today...

- Direction angles and direction cosines
- Projections and work

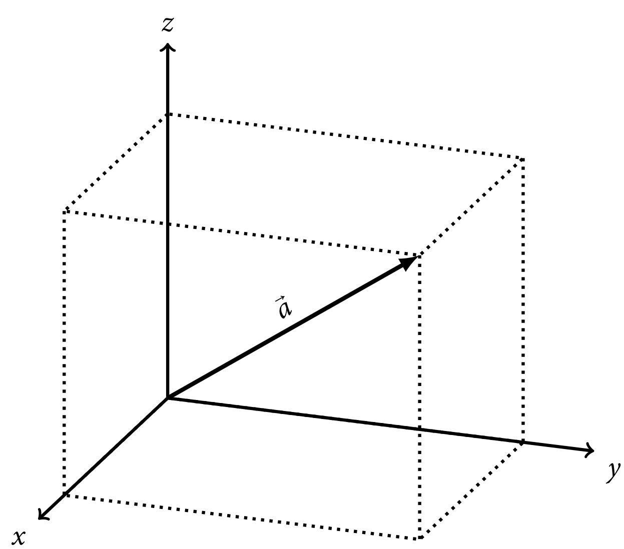
2 Warm up

Example 1. Consider the triangle with vertices $P(2,0)$, $Q(0,3)$ and $R(3,4)$.

(a) Find $\overrightarrow{PQ}$, $\overrightarrow{QR}$ and $\overrightarrow{PR}$.
(b) Find the angle $\angle PQR$ (hint: find the angle between $\overrightarrow{QP}$ and $\overrightarrow{QR}$)

3 Direction angles and direction cosines

- **Direction angles** for vector $\vec{a} = \langle a_1, a_2, a_3 \rangle$:



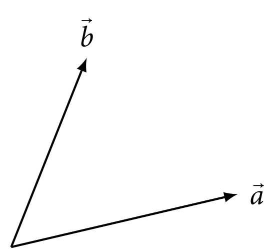
- Again, we take $\alpha, \beta\, \gamma$ always to be in $[0, \pi]$
- Remember that if θ is the angle between $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$

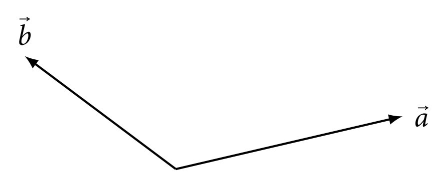
- **Direction cosines**
  - $\cos\alpha =$ $\boxed{}$
  - $\cos\beta =$ $\boxed{}$
  - $\cos\gamma =$ $\boxed{}$
- Note that we can write $\dfrac{1}{|\vec{a}|}\vec{a} =$ $\boxed{}$

Example 2. Find the direction angles of $\vec{a} = \langle 2, 1, 3 \rangle$.

4 Projections

- **Vector projection** of $\vec{b}$ onto $\vec{a}$:



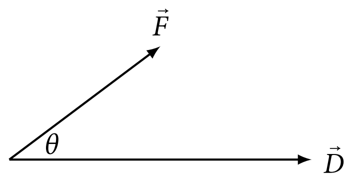


  - Denoted by $\text{proj}_{\vec{a}}\vec{b}$
  - "Shadow" of $\vec{b}$ onto $\vec{a}$
- **Scalar projection** of $\vec{b}$ onto $\vec{a}$ = signed magnitude of $\text{proj}_{\vec{a}}\vec{b}$
  - Also called the **component** of $\vec{b}$ along $\vec{a}$
  - Denoted by $\text{comp}_{\vec{a}}\vec{b}$
- The scalar and vector projections can be computed using dot products:
  - $\text{comp}_{\vec{a}}\vec{b} =$ $\boxed{}$
  - $\text{proj}_{\vec{a}}\vec{b} =$ $\boxed{}$

Example 3. Find the scalar projection and vector projection of $\vec{b} = \langle 1, 1, 2 \rangle$ onto $\vec{a} = \langle -2, 3, 1 \rangle$.

- The **work** done by a constant force $\vec{F}$ in moving an object along a displacement vector $\vec{D}$ is defined as

$$W = (\text{component of } \vec{F} \text{ along } \vec{D})(\text{distance moved})$$



$\Rightarrow W =$

Example 4. A force $\vec{F} = 5\vec{i} - 2\vec{j} + 3\vec{k}$ moves a particle from the point $P(2, 0, -1)$ to the point $Q(6, 2, 4)$. Find the work done.